ON THE POWER OF BALANCE-CHARACTERISTIC FINITE-DIFFERENCE APPROACH FOR CONVECTION-DIFFUSION PROBLEMS

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When solving convection-diffusion equations numerically, with convection dominating, difficulties arise that make the numerical solution of convection-diffusion problems one of the most challenging topics in computational fluid mechanics. It is well known that the Eulerian methods suffer from excessive numerical dispersion and dissipation. This has caused many researchers to abandon traditional continuum mechanics techniques in favour of Lagrangian and Eulerian-Lagrangian methods with mass and interface tracking.

In this talk we will present a new approach to developing explicit high-resolution Eulerian algorithms for convection-diffusion problems. These algorithms are based on introducing two separate variables, which account for transport and conservation respectively and use characteristic upwinding. It is proposed to split the numerical procedure into two parts. At the first stage, linear finite-difference schemes are defined on the most compact special stencil which are devised with improved dissipation and dispersion properties. At the second stage a minimal correction algorithm is applied to enforce the maximum principle. For illustration, some explicit monotonic algorithms with second-order accuracy on smooth solutions are shown which are stable for Courant number less than one. It is demonstrated that the developed algorithms compare favourably with the conventional finite-difference Total Variation Diminishing techniques, which make use of flux limiting.

In the further development of the balance-characteristic approach, it is shown that the numerical algorithms for hyperbolic conservation laws give exact solutions for a linear convection equation with piece-wise linear initial data. Two particular versions of such algorithms, called Jumping Transport Algorithms, are described. The Jumping Transport Algorithms are (i) explicit, hence not required to solve systems of algebraic equations, (ii) make use of the most compact and fixed space-time stencil which only uses variables from one space cell and two adjacent time layers and (iii) conservative and monotonic for Courant number less than one. In particular, the new algorithm is shown to solve the linear convection problem for a grid delta-function exactly and to satisfy the superposition principle. Modifications of Jumping Transport Algorithms for non-uniform grids and convection-diffusion problems are described with the aid of numerical examples.

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