Ellipticity Questions in Soft-Tissue Engineering

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To explain a variety of effects in physiology we require an accurate knowledge of the biological material behaviour on different scales: the nano-, ultra-, micro-, tissue-, and macro-scale. The structural features of the different phases on each level have an influence on the constitutive behaviour at the coarser scales. For soft tissues the collagen-phase can be considered as the basic structural element. In a first approach any kind of history effects which are, however, an intrinsic feature of biological materials are usually neglected. The material behaviour of the collagen network in tension is characterized by an exponential-type behaviour in fiber direction and by a much weaker stress-strain relation in the plane perpendicular to the fibers. In the last decades, the concepts of anisotropic finite elasticity has been used extensively to model biological soft tissues. The nonlinear tensile behaviour is often modelled by exponential type laws, see e.g. FUNG [2], WEISS ET AL. [6], HOLZAPFEL ET AL. [3] and the references therein and SCHRÖDER & NEFF [4]. In order to guarantee the existence of minimizers in finite elasticity, the free energy function has to be quasiconvex. This condition is rather complicated to handle. Therefore, polyconvex functions - which are always quasiconvex and Legendre-Hadamard elliptic, see BALL [1]- are usually considered when we are interested in constructing new energy expressions. An investigation of the Legendre-Hadamard and the strong ellipticity for Fung-type models is presented in WILBER & WALTON [7]. For a systematic treatment of anisotropic, polyconvex free energies within the framework of the invariant theory see SCHRÖDER & NEFF [5], [4]. In this talk we discuss the importants of general convexity conditions and focus on the analysis of several constitutive models for soft tissues and propose new elliptic and polyxconvex, transversely isotropic free energies.

References.

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