EXTENDED FINITE ELEMENT AND LEVEL SET METHODS FOR BIOFILM GROWTH SIMULATIONS

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Although one-dimensional models of biofilms are often adequate for description purposes, multi-dimensional models are required to assess the significant spatial variability of biofilm density, surface shape and microbial activity. In this paper, a two-dimensional quantitative model of biofilm growth is presented. The eXtended Finite Element method is used to account for the presence of the biofilm through discontinuous partition of unity enrichment of the usual finite element approximation space. Thanks to this enrichment procedure, the biofilm can be grown through a fixed mesh, which suppresses the need for remeshing the discontinuity at each step. The geometry of the film is numerically described using a level set method, which tracks the discontinuity surface by solving a hyperbolic conservation equation. The signed-distance function to the zero level set representing the interface is an ideal candidate for a discontinuous enrichment function, which makes the level set method a natural companion to the eXtended Finite Element Method for problems with moving interfaces. Numerical examples are presented to illustrate the method.

References

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