COUPLED "EMPMTH" FEMS FOR TRANSPORT IN SOFT BIOLOGICAL STRUCTURES

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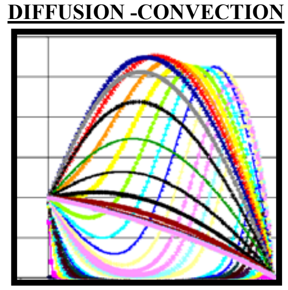
During the last decade, porous media/mixture models have been used to simulate soft hydrated biological structures and tissue-engineered materials. The stresses; fluid pressures; deformations; heat, fluid, and mobile species transport; and biological factors affect growth, remodeling, and cellular homeostasis. Finite element models (FEMs), based upon experiments, can predict these fields to provide an understanding of normal or pathological conditions and optimize prosthetic or sensor designs.

A general "EMPMTH" (Electro-mechano-chemical Mixed Porous Media Transport Heating) continuum theory and FEMs (extension of [1]) are presented including geometric nonlinearity in anisotropic, fluid-saturated porous media (porosity, $n = dV^f / dV$ or $n_0 = dV_0^f / dV_0$; fixed charge density, FCD; $c^F = dn^F / dV^f$ or $\tilde{c}_0^F = dn^F / dV_0$). The response of the incompressible porous solid (*s*) is arbitrary (e.g., fibrous tissue matrix or crushable foam). The fluid (*f*) is incompressible with dissolved mobile charged species ($\alpha, \beta = 1, 2, \ldots, \mathrm{N}_{\mathrm{species}}$). "Primary" fields are displacements, u_i and generalized potentials, $\tilde{v}^{\xi *}$ ($\xi, \eta = f, \alpha, h$). "Secondary" fields include fluid pressure, p^f; electrical potential, $\tilde{\mu}^e$; concentrations, c^α (or $\tilde{c}^\alpha = Jnc^\alpha$); and temperature, $c^h = \theta - \theta_0$. Heat transport (*h*) occurs in the material. The Lagrangian conservation equations are $T_{ji,j} = 0$ and $\tilde{j}^{\xi r}_{k,k} + \tilde{Q}^\xi = 0$ with $\tilde{Q}^f = \dot{J}$, $\tilde{Q}^\alpha = \dot{\tilde{c}}^\alpha$, and $\tilde{Q}^h = \dot{\tilde{c}}^h$. Relative fluxes, $\tilde{j}^{\xi r}_i$: $\tilde{j}^{fr}_i = \tilde{v}^{fr}_i$, $\tilde{j}^{\alpha r}_i = c^\alpha \tilde{v}^{\alpha r}_i$, $\tilde{j}^{hr}_i = c^h \tilde{v}^{hr}_i$, and $\tilde{j}^{er}_i = \tilde{i}^{er}_i = \Sigma_\alpha^{N_{\mathrm{species}}} z^\alpha \tilde{j}^{\alpha r}_i$ (relative electric current). An "effective stress principle" and generalized Darcy law are $\sigma_{ij} = \sigma^{eff}_{ij} - p^f \delta_{ij}$ and $\tilde{j}^{\xi r}_i = -\Sigma_\eta \tilde{L}^{\xi\eta}_{ij} \tilde{v}^{\eta *}_{,j}$; e.g. $S^{eff}_{ij} = J F^{-1}_{ik} \sigma^{eff}_{km} F^{-1}_{jm} = \partial U^{eff} / \partial E_{ij}$ with $U^{eff} = U^{eff}(\varphi)$, $\tilde{L}^{\xi\eta}_{ij} = \tilde{L}^{\xi\eta}_{ij}(\varphi)$, and $\varphi = \varphi(E_{kl}, J, \tilde{v}^{\xi *}, c^h)$ for an extended porohyperelastic Fung formulation. Electro-neutrality is $\Sigma_\alpha (z^\alpha \tilde{c}^\alpha) + \tilde{c}_0^F = 0$. Elemental FEM interpolations (Galerkin formulation) are $\mathbf{u} = \mathbf{N}_u \bar{\mathbf{u}}$, $\mathbf{e} = \mathbf{B}_u \bar{\mathbf{u}}$, $\tilde{\mathbf{v}}^{\xi *} = \mathbf{N}_\xi \bar{\mathbf{v}}^\xi$, and $\tilde{\mathbf{e}}^\xi = \mathbf{B}_\xi \bar{\mathbf{v}}^\xi$ yielding elemental residuals $\boldsymbol{\psi}_\mathbf{u} = \int \bar{\mathbf{B}}_\mathbf{u}^{\mathrm{T}} \mathbf{S}\, dV_0 - \int \mathbf{N}_\mathbf{u}^{\mathrm{T}} \mathbf{t}^{(\boldsymbol{\sigma})} dA = \mathbf{0}$ and $\boldsymbol{\psi}_\xi = -\int \mathbf{N}_\xi^{\mathrm{T}} Q^\xi dV_0 - \int \mathbf{B}_\xi^{\mathrm{T}} \tilde{\mathbf{j}}^{\xi *} dV_0 - \int \mathbf{N}_\xi^{\mathrm{T}} \mathbf{t}^{(\xi)} dA = \mathbf{0}$ that are assembled to global form, boundary and initial conditions imposed, and integrated in time (iterative predictor-corrector algorithms).

DIFFUSION -CONVECTION



Concentration c(X,t)

Results include steady or cyclic porohyperelastic ABAQUS FEMs and EMPMTH FEMs of diffusion-convection in a porohyperelastic solid with finite strains (see right); and electrical, chemical, and mechanically coupled field problems with osmosis and FCD distributions in multi-layered material interfaces. EMPMTH FEMs have applications in biomechanics and tissue engineering (e.g. cardiovascular, orthopedic, and local drug delivery systems) and for analyses of coupled transport in soils and geomechanics.

Acknowledgement NSF Grant BES-9810098.

References

[1] B. R. Simon, G. A. Radtke, P. H. Rigby, S. K. Williams, and Z. P. Liu, "Extended 'LMPHETS' Finite Element Models for Coupled Mechano-Electro-Chemical Transport in Soft Tissues," *ASME-BED, IMECE 2002-32604 CD*, 2002.