ORTHOGONAL REPRESENTATIONS FOR TRANSVERSELY ISOTROPIC HYPERELASTICITY: THEORY AND NUMERICAL ASPECTS

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A number of biotissues can be effectively modeled as transversely isotropic hyperelastic solids. In the analysis of such solids, it is often of interest to represent the strain energy functions in terms of strain invariants that give rise to physically meaningful, and mutually orthogonal, stress terms. Such decompositions have been widely used in the analysis of isotropic hyperelasticity [1, 2]. However, for anisotropic solids, this issue has not been given adequate attention till recently [3].

In this talk, we present theoretical and numerical formulations for transversely isotropic hyperelasticity that give rise to segregated stress terms. We first introduce a stress decomposition, that splits the Cauchy stress into the sum of mutually orthogonal terms consisting of the pressure, the deviatoric tension along the fiber (i.e. the preferred) direction, shear stress along the fiber direction and shear stress in the transverse plane. Then, we introduce a multiplicative decomposition of deformation gradient that leads to a family of strain invariants conjugate to the above stress terms. Last, we develop a computational procedure for the finite element implementation of this constitutive approach. The formulation essentially generalizes the well-developed deviatoric-isochoric decomposition in isotropic hyperelasticity, and carries over the computational structure of the latter. Numerical examples are presented to demonstrate the effectiveness of the new formulation.

References

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