A NON-LINEAR GALERKIN METHOD FOR THE STUDY OF FOKKER-PLANCK EQUATIONS

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Molecular models based on PDE deriving from Fokker-Planck equations are widely used in rheology: the sample consists of a molecule population that is described in terms of a distribution function defined in an adequate space configuration. Brownian motion is usually described with a laplacian operator.
This class of models are often numerically integrated with a linear Galerkin procedure: the distribution function is decomposed in a series of eigenfunctions of the laplacian operator. This mathematical procedure leads to an infinite set of ODEs in terms of the series coefficients. In order to end up with a closed set of equations, the infinite dimensional system is truncated to a specific order that ensures a quantitative description of the solution.
Here, we adopt a non-linear Galerkin procedure based on the Inertial Manifolds theory [1]. This approach assumes that long-term dynamics of dissipative differential equations is influenced only by a finite number of low-order harmonics whereas higher harmonics can be estimated directly from lower ones [2]. In this way, a quantitative description of the dynamics can be obtained with a limited set of state variables, thus resulting in a lowering of the computational time.
The rigid-rod model for liquid crystalline polymers is considered as an example [3-4]. The dynamics of the model is analyzed in shearing flow conditions, which are characterized by a rich dynamics. A two dimensional version of the model is here considered, as the molecules are constrained in the flow plane. In this case, the proper eigenfuctions are the standard Fourier series [5].
The non linear Galerkin results are compared with corresponding solutions obtained with the traditional linear Galerkin scheme. It turns out that the non linear procedure leads to a significant reduction of the required harmonics with respect to the traditional approach. The comparison of the two techniques has been performed also with the aid of bifurcation diagrams.

References

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