A CORRECTED COMPLETED DOUBLE LAYER BOUNDARY INTEGRAL EQUATION

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The completed double layer boundary integral equation (CDL–BIE) has been used extensively to model the mobility problem in suspensions of rigid particles. However, it has been shown that the method breaks down at moderate to high particle volume fractions, when particle surfaces become excessively close to one another. While theoretically possible, improving the solution by mesh refinement is impractical because of the slow convergence of the solution combined with the increasingly ill-conditioned matrices that result. Recently, analytic lubrication solutions describing the forces arising from the relative motion between two particles have been used to correct the boundary integral equation [1]. This involves a small modification to the surface of particles neighboring a particle where collocation is taking place, to such an extent that the numerical solution of the BIE is accurate. The lubrication forces and torques resulting from the modified geometry are removed, and the forces and torques corresponding to the original geometry are applied.

The main difficulty found in this approach lies in the evaluation of the lubrication forces, which are not known *a priori*. These are a linear function of the particle velocities, and through a series of linear transformations and rotations, they can be incorporated into the linear system that results from the discretization of the boundary integral equation. However, the modification to the linear system disrupts the diagonal dominance inherent in second–kind integral formulations, negating the gain in efficiency that results from the utilization of iterative solvers. The application of suitable preconditioners, based on the structure of the matrix, can restore the diagonally dominant nature of the systems.

The building blocks of the preconditioner can be constructed locally. However, the preconditioner requires the inversion of a matrix the size of which is proportional to the number of particles in the system. The matrix is evaluated locally in row–wise blocks. If the number of particles is moderate the inverse can be evaluated serially on one process, and the inverse can be subsequently re–distributed to individual processes. For large numbers of particles, parallel evaluation may be required.

The preconditioned corrected CDL–BIE is applied to a number of test cases. The accuracy of the solution when particle surfaces are very close is improved dramatically, by several orders of magnitude. In particular, it becomes unnecessary to produce very fine discretizations in order to obtain acceptable accuracy. This results in smaller systems with better condition numbers, where the convergence rate is relatively independent of problem size. In terms of performance, measured by the ratio of accuracy to computational expense, the corrected CDL–BIE is beyond comparison to the uncorrected method.

References

[1] A. Mammoli, "Towards a reliable method for predicting the properties of multiphase fluids," *Engineering Analysis with Boundary Elements*, v. 26, p. 861-870, 2002.