ELASTO-PLASTIC CONTACT OF FRACTAL SURFACES

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Measurements of rough surfaces show that the roughness topography can be described as fractal over several length scales. A suitable description is then given by the discrete structure function defined as

$$S(x_k) = \frac{1}{N-k} \sum_{i=1}^{N-k} (z_{i+k} - z_i)^2 \qquad k = 1, \ldots, N-1 \; . \tag{1}$$

For a large class of typical surfaces measured structure functions can be approximated by

$$S(x_k) = 2\sigma_z^2 \left\{ 1 - \exp\left[- \left(x_k / x_{\mathrm{T}} \right)^{4-2D} \right] \right\} \; , \tag{2}$$

where σ_z is the rms-value of the roughness, x_{T} is a typical distance, which indicates the transition between fractal behaviour at high wavenumbers and stationary behaviour at low wavenumbers, and D is the fractal dimension in the fractal region.

To study the normal contact behaviour of such fractal surfaces numerically, the authors present first a spectral method to generate fractal surfaces according to the structure function defined above. In a second step the surfaces are used in a contact simulation employing elastic halfspace theory. Results of these simulations agree well with similar calculations performed by Borri-Brunetto et al. [1]. Using different wavelength resolutions λ_{lim} of the generated surfaces, the pressure-gap laws can be shown to converge once the resolution is fine enough. However, the corresponding real area of contact decreases for finer resolutions, indicating that for purely elastic contact the real area is a fractal surface with dimension $D < 2$. Such a behaviour seems unrealistic from a physical point of view, since the decrease and fragmentation of the real area of contact will lead to very high local pressures and thus to plastic deformation. Therefore the authors propose an elasto-plastic theory, which leads to a finite area of contact, given by the simple relation

$$A_r / A_0 = p / p_{\mathrm{max}} \; . \tag{3}$$

The results allow to draw the following conclusions, see also [2]:
• To calculate the pressure-gap relationship, the wavelength limit λ_{lim} has to be chosen smaller than the transition length x_{T}, which can be determined from a plot of the structure function. A reasonable wavelength limit would be $\lambda_{\mathrm{lim}} \approx x_{\mathrm{T}}/10 \ldots x_{\mathrm{T}}/5$.
• Upon initial contact all fractal surfaces deform plastically, resulting in a simple area-pressure relationship of the form (3).

References

[1] M. Borri-Brunetto, A. Carpinteri, and B. Chiaia, "Lacunarity of the Contact Domain Betweeen Elastic Bodies with Rough Boundaries," G. Frantziskonis (Ed.), *: PROBAMAT – 21st Century: Probabilities and Materials*, p 45-66, Dordrecht: Kluwer, 1998.
[2] K. Willner, "Elasto-Plastic Normal Contact of 3D Fractal Surfaces Using Halfspace Theory," to appear in *Journal of Tribology*.