A NEW APPROACH TO OPTIMAL SHAPE DESIGN OF FRICTIONAL CONTACT PROBLEMS USING VARIATIONAL INEQUALITIES

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Contact problems are inherently nonlinear. With the friction forces typically being modeled by nonlinear relationships of the Coulomb's type and contact surfaces being unknown a priori, accurate modeling of contact problems can be burdensome. In this regard, contact problems are formulated using a more rigorous and consistent method, known as the variational inequalities (VI) approach to eliminate most of the mathematical inconsistencies present in the traditional variational methods. In order to minimize contact stresses, the contact potential energy is used as the optimization objective function. Objective functions of contact problems governed by variational inequalities are, however, nonsmooth which lead to local minima.

Currently, two techniques are being employed to solve such problems. The first is based on the use of predefined frictional forces [1], which does not represent the actual interface behavior. Recently, Kim et al. [2] developed a sensitivity analysis based on the use of penalty method to impose the kinematic contact conditions in both the normal and tangential directions. This can lead to a smooth optimization problem. However, interpenetration has been reported in the final optimal design. In this study, a new approach is developed to minimize contact stresses, using variational inequalities.

Three aspects of the work are, accordingly, examined. The first is concerned with the modeling of contact surfaces using rational cubic splines. The control parameters of these splines are used as the design variables. In addition, they should preserve C^1-continuty during the imposition of kinematic contact constrains in order to have local support of the splines. The second involves the development of a novel sensitivity analysis based on the use of the material derivative approach. The solution of the resulting sensitivity variational inequality provides the material derivative of the contact solution necessary to update the optimization design variables. Finally, the resulting nonsmooth optimization problem is solved using Proximal Bundle method. In this case, the subgradients are gathered in a bundle to provide the necessary information to obtain a descent direction. A number of verification examples are solved to show the accuracy and convergence rate of the developed approach.

References

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