ADVANCES IN THE TIME INTEGRATION OF DYNAMICAL SYSTEMS. APPLICATIONS TO CRASHWORTHINESS PROBLEMS.

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One can resort to two families of algorithms to integrate the equations of evolution of dynamical systems: the implicit family and the explicit family. In this paper, we focus on the implicit family. The most widely used implicit algorithm is the Newmark algorithm. Nevertheless, the total energy of a dynamical system, whose evolution equations are integrated by this algorithm, generally exhibits oscillations in time, even if the amplitude of these oscillations is limited for linear systems. But for non-linear models, larger instabilities can arise, leading to divergence of the numerical simulation. Moreover, the angular momentum is not conserved between two time stations. To avoid divergence due to the numerical instabilities, numerical damping was introduced, leading to the generalized-α methods. Therefore a new kind of dynamics integration algorithms has appeared that verifies the mechanical laws of conservation (i.e. conservation of linear momentum, angular momentum and total energy) and that remains stable in the non-linear range. The first algorithm verifying these properties was described by Simo and Tarnow [1]. They called this algorithm Energy Momentum Conserving Algorithms or EMCA. It consists in a mid-point scheme with an adequate evaluation of the internal forces. This adequate evaluation was given for a Saint Venant-Kirchhoff hyperelastic material. A generalization to other hyperelastic models was given by Laursen [2], who iteratively solves a new equation for each Gauss point to determine the adequate second Piola-Kirchhoff stress tensor. Another solution that avoids this iterative procedure leads to a general formulation of the second Piola-Kirchhoff stress tensor, as given by Gonzalez [3]. This formulation is valid for general hyperelastic materials. The EMCA was recently extended to dynamic finite deformation plasticity by Meng and Laursen [4].

All the conserving methods described above were established for hyperelastic materials. We have recently established a new expression of the internal forces for the hypoelastic materials using the final rotation scheme. When associated with the mid-point scheme, this expression ensure the conservation laws of the mechanics for a hypoelastic constitutive model. Moreover, using the radial return mapping, we prove that this adaptation remains consistent with the Druker postulate when plastic deformation occurs. Numerical examples presented will prove that this newly developed algorithm conserves energy and momentum both for academic examples and more realistic crash simulations.

References

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