

# A DISCONTINUOUS GALERKIN METHOD FOR THREE-DIMENSIONAL SHALLOW WATER EQUATIONS

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One has to note that most existing numerical methods for the shallow-water equations (SWE) have serious drawbacks with regard to stability, local conservation, and ability to accommodate hp-adaptivity and parallel implementation. These problems become even more evident if we try to simulate systems involving discontinuities, shock waves, etc.

The local discontinuous Galerkin (LDG) method, on the contrary, offers unique advantages in terms of flexibility and stability control. Though demonstrated to work well for the 2D SWE (see e.g. [2]) as well as for compressible Navier-Stokes equations in 3D, at this time there is not much experience implementing the LDG method for problems with a free surface, in particular, if the free surface is also approximated using a discontinuous space. Preliminary results obtained from application of the proposed scheme to several test problems indicate excellent stability and accuracy properties providing we use a special numerical flux formulation on the inter-element boundaries.

In our talk, we will discuss theoretical and practical issues arising from application of the LDG method to the discretization of a system of 3D SWE. In the implementation part, we will show numerical results demonstrating performance of our scheme on a range of 3D problems.

## References

- [1] B. Cockburn, C.-W. Shu, “The local discontinuous Galerkin finite element method for convection-diffusion systems”, *SIAM J. Numer. Anal.*, v 35, p. 2440-2463, 1998.
- [2] V. Aizinger, C. Dawson, “ A discontinuous Galerkin method for two-dimensional flow and transport in shallow water,” *Advances in Water Resources*, v. 25, p. 67-84, 2002.