A LOCAL DISCONTINUOUS GALERKIN FORMULATION FOR THE CLASS OF ELASTIC WAVE PROPAGATION PROBLEMS[1]

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The presence of spurious oscillations which result from the spatial discretization of the governing equations in many transient applications such as elastodynamics analysis is not a desirable feature. Several time discretization methods, usually embed numerical damping, have been employed to annihilate these oscillations. However, they inherit disadvantages such as dissipation of energy that is not suitable for long term analysis and the need for considerable insight to the choice of damping. Hughes et. al [2] developed a time-discontinuous Galerkin procedure for the second-order hyperbolic problems. Although, the method retains some of the attractive features of the fully-discontinuous method for the first-order problems, local conservation property is not preserved [3]. Unlike space-time formulations [3], in this paper we present the local discontinuous Galerkin (LDG) formulations of semi-discretization for second-order elastodynamics.

The following second-order elastodynamics initial-boundary value problem is considered. Let a linear elastic solid occupying a region $\Omega \in \Re^n$ with $\partial\Omega$. The balance of linear momentum reads $\rho\ddot{\mathbf{u}} - \nabla.\boldsymbol{\sigma} = \mathbf{f}$ in Ω. The body is acted upon a body forces $\mathbf{f} : \Omega \times [0, T) \rightarrow \Re^n$, prescribed surface tractions $\bar{\mathbf{t}} : \Gamma_N \times [0, T) \rightarrow \Re^n$, prescribed displacement $\bar{\mathbf{u}} : \Gamma_D \times [0, T) \rightarrow \Re^n$, with initial conditions $\mathbf{u} : \Omega \times 0 \rightarrow \Re^n$ and $\mathbf{v} : \Omega \times 0 \rightarrow \Re^n$ where $\Gamma_D \cup \Gamma_N = \partial\Omega$, $\Gamma_D \cap \Gamma_N = \emptyset$, and Γ_D can have zero measure. The stress $\boldsymbol{\sigma}$ and the strain $\boldsymbol{\epsilon}$ are defined by $\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\epsilon}$. A critical aspect of the formulation requires the choice of the unknown variables. For the case of the elastodynamics problem, appropriate first-order form of representations are obtained when the above are expressed in terms of velocity and stress since discontinuities manifest in these variables for elastic wave propagation problems. Letting $\dot{\mathbf{u}} = \mathbf{v}$ and with modification of constitutive equations into the rate form, yields the following set of equations $\rho\dot{\mathbf{v}} - \nabla.\boldsymbol{\sigma} = \mathbf{f}$, with $\dot{\boldsymbol{\sigma}} = \mathbf{C} : \nabla\mathbf{v}$ subjected to prescribed $\bar{\mathbf{t}}$, prescribed velocity $\bar{\mathbf{v}} : \Gamma_D \times [0, T) \rightarrow \Re^n$, and with initial conditions $\dot{\mathbf{u}} : \Omega \times 0 \rightarrow \Re^n$ and $\dot{\boldsymbol{\sigma}} : \Omega \times 0 \rightarrow \Re^n$. We seek a LDG [1] formulation that is amenable to both space and time discontinuous (unlike space-time [3]) discretization, that inherits the block diagonal mass matrix structure and thereby allowing element-by-element computational procedure. The key to the formulation is that one allows for jumps in the field variables $(\mathbf{v}, \boldsymbol{\sigma})$ across inter space element boundaries with proper choice of conditions across it. The results are verified from numerical solutions to wave propagation in which jump in stress and velocity are prescribed at the wavefront by applying a step load to a one-dimensional linear homogeneous/heterogeneous elastic bar. In addition, the dispersion and the dissipation properties of the DG method are performed.

References

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