

# A MODIFICATION OF THE CROUZEIX-RAVIART ELEMENT VALID FOR NON-LINEAR FINITE DEFORMATION SOLID MECHANICS

J. M. Solberg<sup>a</sup> and M. A. Puso<sup>b</sup>

<sup>a</sup>Methods Development Group  
Mail Stop L-125  
Lawrence Livermore National Laboratory  
7000 East Avenue  
Livermore, CA 94550-9234  
solberg2@llnl.gov

<sup>b</sup>Methods Development Group  
Mail Stop L-125  
Lawrence Livermore National Laboratory  
7000 East Avenue  
Livermore, CA 94550-9234  
puso1@llnl.gov

The 4-node quadrilateral in two-dimensions and the 8-node hexahedron in 3-dimensions are the favored tools for the finite element analysis of solid mechanics problems where full- or near-incompressibility, finite deformation, and/or non-linear material models are present. Such techniques as hourglass control allow these elements to model phenomena with a high ratio of accuracy and stability to expense.

Unfortunately, the creation of adequate quadrilateral/hexahedral meshes generally requires significant and tedious user intervention. Alternatively, low order triangular/tetrahedral meshes (3-node triangles/4-node tetrahedra) may be employed. It is well known however that these elements may exhibit seriously degraded accuracy, especially in cases involving significant bending and/or near incompressibility, where such elements are subject to severe locking.

On the other hand, the linear, partially discontinuous Crouzeix-Raviart element does not lock, but suffers from an inability to repress certain local rigid-body motions<sup>1</sup>. Recent work has established a method to stabilize this element<sup>2</sup>. However, this work is formulated on the basis of linear elasticity, with an additional penalty term. It is unclear how to extend this work generally to problems involving finite rotations and/or non-linear material models.

In this work, the idea of the lifting function from Arnold, et. al<sup>3</sup> has been applied to the Crouzeix-Raviart element, producing an element which is stable and does not lock. The lifting function has been utilized to produce additional strain fields, which provide a natural means to extend the model to finite deformations and non-linear material models. The model may be tuned to provide good behavior in bending. Numerical examples are presented and future work is discussed.

## References

- [1] D. N. Arnold, "On Nonconforming Linear-Constant Elements for Some Variants of the Stokes Equations", Note presented in honor of Franco Brezzi, June 24th, 1993, Instituto di Analisi Numerica del C.N.R., Pavia, Italy.
- [2] P. Hansbo and M. G. Larson, "Discontinuous Galerkin and the Crouzeix-Raviart Element: Application to Elasticity", Chalmers Finite Element Center Preprints, 2000-09, Goteborg, Sweden, 2001.
- [3] D. N. Arnold, F. Brezzi, B. Cockburn, and L. D. Marin "Unified Analysis of Discontinuous Galerkin Methods for Elliptic Problems", *SIAM Journal of Numerical Analysis*, v. 39, no. 5, p. 1749-1779, 2002.