OPTIMAL BV ESTIMATES FOR A DG METHOD FOR LINEAR ELASTICITY

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We are interested in a Discontinuous Galerkin (DG) method for studying the mechanical behavior of solids. We begin by analyzing the linear elasticity problem, with an eye toward a formulation for nonlinear elastic-plastic problems and cohesive elements. There are several benefits of such an approach, including the potential for efficient hp-adaptivity, for example, using meshes with hanging nodes with adaptive mesh refinement, and the prospect of rigorously handling problems with discontinuous displacements as arise in the study of fracture. Our discrete formulation derives from the Hellinger-Reissner variational principle with the addition of stabilization terms analogous to those previously considered by Bassi and Rebay [1] for the Navier-Stokes equations and by Brezzi, *et al.,* [2] for the scalar Poisson equation. The result is a symmetric bilinear form.

We will present a derivation of the discrete equations based on a discrete variational principle and then discuss convergence properties of the method. Error estimates for discontinuous Galerkin methods are usually obtained in terms of mesh-dependent norms. We will derive optimal convergence rates in a mesh-dependent norm that mimics the analysis in [2] of the scalar Poisson equation. The classical analysis of the equations of linear elasticity needs a global version of Korn's first inequality to insure coerciveness of the bilinear form. In contrast, we will show a generalization of Korn's second inequality on the element level which allows us to obtain an improved mesh-dependent estimate. This result then allows us to show uniform convergence in the mesh-independent BV norm. Convergence rates will be demonstrated on one-dimensional and two-dimensional examples.

References

[1] F. Bassi and S. Rebay, "' High-Order Accurate Discontinuous Finite Element Method for the Numerical Solution of the Compressible Navier-Stokes Equations", *J. Comput. Phys.*, v. 131 pp. 267–279, 1997.
[2] F. Brezzi , M. Manzini, D. Marini, P. Pietra and A. Russo, "'Discontinuous finite elements for diffusion problems", *Atti Convegno in onore di F. Brioschi (Milano 1997), Istituto Lombardo, Accademia di Scienze e Lettere*, pp. 197-217, 2001.