ADAPTIVE AND PARALLEL DISCONTINUOUS GALERKIN METHODS FOR HYPERBOLIC CONSERVATION LAWS

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We describe several theoretical and computational aspects of the discontinuous Galerkin (DG) method as it applies to hyperbolic conservation laws. Our focus is on the Euler equations of compressible, inviscid fluid dynamics; however, the method is capable of handling virtually any hyperbolic system. We utilize an orthogonal (Dubiner) basis on triangles and tetrahedra that diagonalizes the mass matrix. We describe several numerical flux functions and explore their perfomance on model compressible flow problems. Spurious oscillations in the solution arise when high-order methods are applied to problems with discontinuities. Low-order methods, on the other hand, introduce excessive diffusion. With DG methods, the strategy for controlling spurious oscillation is to limit variations in the solution and/or flux. We describe several limiting strategies, which, unfortunately, do not preserve a high order of accuracy near smooth extrema. We develop a discontinuity detection strategy that is based on jumps in certain quantities across element boundaries that identifies regions containing discontinuities. With detection, limiting need only be applied in the vicinity of discontinuities and, thus, high-order accuracy can be preserved elsewhere. We further describe computational environments and data structures for serial and parallel adaptive computation. We present adaptive h- and p-refinement procedures on structured and unstructured meshes that operate both in space and time. Local time step refinement, in particular, can dramatically improve performance on unstructured meshes. Spatial adaptivity may be done so as to align the mesh with evolving solution. In this manner, the mesh may be differentially graded in directions normal to discontinuities. We illustrate the performance of the DG software using several transient compressible flow problems, including Rayleigh-Taylor and Kelvin-Helmholtz instabilities and blast.