ANALYSIS OF A SPACETIME DISCONTINUOUS GALERKIN METHOD FOR SYSTEMS OF CONSERVATION LAWS[1]

K. Jegdic[a], **R. Jerrard**[a] and **R. B. Haber**[b]

[a]Department of Mathematics
[b]Department of Theoretical and Applied Mechanics

Center for Process Simulation and Design
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801
{jegdic, rjerrard, r-haber}@uiuc.edu

We present a mathematical analysis of a spacetime discontinuous Galerkin (SDG) method for systems of nonlinear hyperbolic conservation laws (described by Palaniappan et al. elsewhere in this proceedings). The method is motivated by the physical properties of such systems, and can be used on either structured or unstructured spacetime grids. When the mesh satisfies a certain causality cone constraint, a direct element-by-element solution procedure is possible. The mathematical analysis is complicated by the fact that the SDG formulation does not include any stabilization terms, by its spacetime format, and by the nonlinearity of the flux function; many of the techniques used to analyze other discontinuous Galerkin methods are not applicable here.

In view of these analytical challenges, we introduce some simplifying assumptions. First, we restrict our attention to systems of two equations whose shock and rarefaction wave curves coincide, as introduced by Temple in [1], and we assume that all element interfaces satisfy the causality cone constraint. Further, we assume that the spatial fluxes are Lipschitz continuous and that the discrete Galerkin space consists of piecewise constant polynomials. Under these assumptions, we show that an approximate SDG solution exists and that it is unique; this ensures that the SDG method is well-defined. The key step in the convergence proof is to show that the Riemann invariants of the approximate solution on each spacetime element are bounded from below and above by the Riemann invariants of the element's inflow data. This result implies the TVD (total variation diminishing) property. As a consequence, the sequence of approximate solutions is precompact, and a convergent subsequence can be extracted. We use this finding to show that the SDG method converges to a weak continuum solution of the Temple system.

Following the solution of a Riemann problem for Temple class systems in [2], we describe a possible generalization of our analysis to the case in which the spacetime mesh contains some coupling boundaries that do not satisfy the cone constraint and across which conservation is defined in terms of the physical Godunov fluxes.

References

[1] B. Temple, "Systems of conservation laws with invariant submanifolds," *Trans. Amer. Math. Soc.*, v. 280, p. 781-795, 1983.
[2] D. Serre, "Solutions a variations bornees pour certains systems hyperboliques de lois de conservation," *J. Diff. Eqns.*, v. 67, p. 137-168, 1987.

[1]Supported by NSF grants DMS 98-73945 and DMR 01-21695.