THE DISCONTINUOUS PETROV-GALERKIN FINITE ELEMENT METHOD: ERROR ANALYSIS AND COMPUTER IMPLEMENTATION

P. Causin[a] and **R. Sacco**[b]

[a] INRIA Rocquencourt - B.P.105
F-78153 Le Chesnay Cedex, France
paola.causin@inria.fr

[b] MOX - Modeling and Scientific Computing
Dipartimento di Matematica "F. Brioschi"
Politecnico di Milano
via Bonardi 9, 20133 Milano, Italy
riccardo.sacco@polimi.it

In this communication we address the error analysis and the computer implementation of the Discontinuous Petrov-Galerkin finite element method (DPG) introduced in ref. [1] and recently investigated in ref. [2]. The most relevant feature of the DPG formulation is the presence of internal and interface variables, the former being totally discontinuous across neighboring elements, the latter being solely defined over the element boundaries. We first clarify connections and relationships between the DPG formulation and the standard Mixed-Hybrid (MH) and Discontinuous Galerkin (DG) methods. Then, we derive the appropriate weak formulation of the DPG method and prove existence and uniqueness of the solution. As for the discretization, we carry out the stability and a priori error analysis of the DPG finite element method, proving in particular that the interface variables enjoy a higher convergence rate than the internal variables as typical of MH methods. The simultaneous presence of internal and interface variables provides furthermore an immediate strategy for devising an automatic grid adaption process driven by an a posteriori error estimator. Some preliminary results on this issue are provided. Eventually, we furnish details about the computer implementation of the method, showing that the DPG method can be fully regarded (and numerically implemented) as a standard DG formulation. Numerical results are presented to validate the convergence performance of the scheme, with particular emphasis on the superconvergence and conservation properties enjoyed by the hybrid variables.

References

[1] C. L. Bottasso, S. Micheletti, R. Sacco, "The Discontinuous Petrov-Galerkin Method for Elliptic Problems", *Comput. Methods Appl. Mech. Engrg.*, v. 191, p. 3391–3409, 2002.

[2] P. Causin, R. Sacco "A Discontinuous Petrov-Galerkin Method with Lagrangian Multipliers for Second Order Elliptic Problems", MOX-Report No.19-May 2003, Politecnico di Milano.