

# AN AUTO-WAVE SOLIDIFICATION PROCESS IN HETEROGENEOUS MEDIA

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Processes of solidification are usually characterized by a number of instabilities of various physical nature. The paper presents the theory, which allows determining conditions of the instability of steady regimes of crystallization of melt saturated by gas. The instability is caused by a formation of a mushy (two-phase) zone close to the crystallization front. The physical mechanism of instability and a formation of wave regime look like as follows. Usually gas solutes better in a liquid phase than in the solid one. Thus propagating crystallization front drives gas soluted in a liquid phase as any admixture with the distribution coefficient less than unity.

When the gas concentration before the front increases up to a certain limiting value, this causes the growth of gas nuclear. These bubbles are caught by the moving front and become defects of ingot. The instability of a steady process is caused by a strong dependence of the nucleation rate upon the driving force of crystallization. Thereby, the growth of nuclear may, under certain specific conditions, lead to a decrease in the metastability (that is, of the transient gas supersaturation). Such a decrease brings about a sharp decrease in the nucleation rate, and, consequently, to a decrease of bubbles forming in melt. In due time propagating supersaturation before the propagating crystallization front again increases, which gives rise to a new enhancement of the nucleation and bubble growth rates. This reduces the supersaturation again and thus an auto-wave regime is established. The auto-wave process can be established even under isothermal conditions. The result of such regime is ingot in which gas bubbles are located "layer by layer".

The following assumptions are put forward in order to develop a mathematical model. We assume the crystallization process to be isothermal and to be limited by diffusion of gas soluted in melt. Diffusion in the solid phase is considered to be negligible and the physical characteristics of the melt to be independent on the concentration. Convection in melt is assumed to be negligible. We suppose the process to be stationary with respect to the crystallization front rate. The crystallization front is assumed to be plane. The concentration of gas in melt is supposed to be dependent only on coordinate normal to the front.

The model of crystallization is based on the equation of diffusion of gas soluted in melt with the boundary condition reflecting the balance of admixture on the front together with the population balance equation expressed in terms of the bubble size distribution. The bubble growth rate depends upon the bubble size and the supersaturation and usually can be represented as a product of two functions depending solely on the radius and supersaturation.

As a result of several modifications we come to the following equation:

$$\frac{\partial u}{\partial \tau} - \frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} - \frac{4\pi\rho\beta\beta_0 D^2}{C_0 v^4} \int_0^\infty f(x', s, \tau') r^2(s) ds.$$

This equation determines completely the evolution of both the polydisperse system of bubbles and the supersaturation for an arbitrary nucleation and bubble growth kinetics.

The theory developed on this basis provides one with a convenient tool to determine conditions under which steady regimes of solidification lose their stability with respect to occasional small disturbances. The neutral stability curves and the corresponding oscillation period can be expressed in terms of relevant physical, chemical and processing parameters for various kinetics of nucleation and bubble growth as well as for different external conditions.