STATISTICAL DISLOCATION DYNAMICS

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The fact that dislocations in a deforming crystal represent a system with many degrees of freedom makes possible the application of the powerful concepts of statistical mechanics to study the collective behavior of dislocations. Indeed, two complementary statistical approaches are currently under development. The first approach is based on direct numerical simulation of discrete dislocation systems, which is termed "dislocation dynamics simulation," and the second approach is a formal one, which is structured along the lines of the "classical kinetic theory." We focus on the latter approach in this lecture because it provides the formal theoretical underpinning of statistical treatment of dislocation ensembles, and call this approach "statistical dislocation dynamics."

This lecture highlights the latest theoretical and numerical developments of 3-D statistical dislocation dynamics: the development of a finite-deformation kinetic description of dislocations and formal definition of statistical dislocation ensembles. In this kinetic description, dislocations are represented by a set of segment-orientation-dependent phase densities in multiple phase spaces, one per slip system, each of which shares the Euclidean crystal space, segment velocity and orientation. The mathematical structure of the theory is based on two sets of governing equations: kinetic equations for the transport and reactions, plus gauge conditions. The kinetic equations are similar to those for point particles, but include reaction and cross slip terms that depend on the line-orientation of dislocations. The gauge conditions account for the linearity of dislocations and have no counterpart in classical kinetic theory of point particles. The kinetic description is complemented by the finite-deformation kinematics, and hence two alternative forms of the kinetic equations are derived, in the referential and spatial crystal configurations. The driving force for evolution is found from the elastic stress field.

The concept of statistical mechanics ensemble is also extended to dynamic dislocation systems by formally defining the null-A ensemble, an ensemble characterized by a null average dislocation density tensor. This ensemble can be used to formally study a dislocation system interacting with a crystal ambient, more or less like the classical canonical ensemble.

Special cases and numerical examples are given to explain various aspects of the developed theory and the connection with discrete simulations is also explained.