THE METHOD OF NEARBY PROBLEMS: AN APPROACH FOR ASSESSING ERROR ESTIMATORS

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The process of discretizing a set of partial differential equations into an algebraic set of equations that can be numerically solved always involves an acknowledged error, termed discretization error. Estimates of the size of the discretization error can be used for mesh adaptation, quantifying error in predictive simulations, and as an integral part of validation simulations. Discretization error estimators can be broadly classified into three categories: 1) extrapolation-based, 2) residual-based, and 3) recovery-based error estimators. The concepts behind extrapolation-based error estimators are derived from Richardson extrapolation, [1] where the numerical solutions on two meshes are extrapolated to zero element size to approximate the exact solution. The residual-based error estimators were initially developed for finite element formulations and include both standard *a posteriori* and adjoint-based error estimators. The recovery-based error estimators were also initially developed for finite elements, and have recently been extended to the finite-volume approach. [2]

A methodology will be presented for generating exact solutions to equations that are “near” the Navier-Stokes equations. [3] First, a highly accurate numerical solution to the Navier-Stokes equations is computed. Second, an analytic function is fit to the numerical solution by least squares optimization. Next, this analytic solution is operated on by the Navier-Stokes equations (including auxiliary relations) to obtain a small analytic source term. When the Navier-Stokes equations are perturbed by adding this source term, the analytic function is recovered as the exact solution. Approaches will be presented which address the “goodness” of the curve-fitting procedure and the “nearness” of the modified set of equations to the Navier-Stokes equations. Examples will be given for compressible fluid flow including fully developed flow in a channel and lid-driven cavity flow. The channel flow is fully captured by a third-order polynomial fit, while the driven-cavity solution is not adequately represented by polynomial curve fits up to fourth order. Alternative basis functions for the curve fitting procedure will be examined in the talk. The generation of an exact solution to a set of equations near the Navier-Stokes equations allows for the evaluation of various discretization error estimators, without reverting to simplification of the governing equations or use of a highly refined “truth” mesh.

References

[1] L. F. Richardson, “The Deferred Approach to the Limit,” *Transaction of the Royal Society of London*, Ser. A 226, pp. 229-361, 1927.
[2] T. J. Barth, and M. G. Larson, “A Posteriori Error Estimates for Higher Order Godunov Finite Volume Methods on Unstructured Meshes,” NASA TR NAS-02-001, February 2002.
[3] C. J. Roy and M. M. Hopkins, “Discretization Error Estimates using Exact Solutions to Nearby Problems,” AIAA Paper 2003-0629, Jan. 2003.