OPERATOR RECOVERY ERROR SOURCE DETECTOR

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The necessity for a reliable measure of the discretization error arises in adaptive mesh refinement (AMR) and in moving mesh adaptation (MMA). Typically, regions of interests are detected in one of two approaches. First, heuristic measures can be devised in a problem dependent fashion based on the intuition of the physical evolution of the system. This approach is often used by experienced developers but makes the use of AMR or MMA more an art than a science. Second, a posteriori error estimators can be used to measure rigorously the truncation error. Such approach is usually highly mathematical and its rigorous application is of limited scope. Nevertheless, the approach can be applied approximately, regardless of it rigor, in a number of applications.

In the present paper we follow the second approach and attempt to find a general detector of the regions where the truncation error is larger. Our aim is to derive a general approach that can be applied to any given set of equations without requiring any a priori knowledge of the physics of the system under investigation or any property of the solution being found. We present an approach [1] based on a generalization of the gradient recovery error estimator. The method is called operator recovery error source detector (ORESD). In the present work we present the technique in its simplest formulation and we apply it to tutorial examples and to often used benchmarks. We consider also the application of the ORESD technique to MMA [2].

The main features of the ORESD are: First, the technique is automatic and does not require any user intervention or any a priori knowledge of the solution or its properties. Second, the ORESD is an a posteriori error indicator, but it is shown to be consistent with the a priori error provided by the modified equation approach. Third, the technique is based on the operators being solved and is tailored to the specific problem at hand. Four, the technique is simple and is based on a small stencil, resulting in a very inexpensive error detection.

In the present work, the ORESD is derived and applied to two tutorial examples: divergence and gradient. With the aid of the two examples and using the general derivation, the ORESD is then applied to the gas dynamics equations. Two benchmarks are used to test the performance. First, a shock tube problem is solved (Sod's shock tube benchmark) in a Lagrangian and in a Eulerian frame. Second, the Colella's wedge problem is solved using CLAWPACK. Finally, the ORESD is applied to the 2D Poisson equation on a uniform and on a non uniform grid to test the application to elliptic problems.

In all examples the operator recovery error source detector succeeds in detecting the real sources of error. The results prove that the ORESD can be used as a general purpose error detector. The detector can be applied to any existing numerical scheme and to any existing code either to adapt the grid in MMA schemes or to refine it in AMR schemes.

References

[1] G. Lapenta, A Recipe to Detect the Error in Discretization Schemes, *Int. J. Num. Meth. Engng.*, to appear.

[2] G. Lapenta, Variational Grid Adaptation Based on the Minimization of Local Truncation Error: Time Independent Problems, *J. Comput. Phys.*, submitted (2002)