

# A POSTERIORI ERROR ESTIMATION AND SUPERCONVERGENCE FOR HYPERBOLIC AND PARABOLIC SYSTEMS

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We develop *A posteriori* error estimates for hyperbolic and parabolic problems that are based on superconvergence properties where (smooth) finite element solutions converge at a higher rate at boundaries or in interiors than they do globally. For parabolic problems, we show that errors of odd-degree polynomial solutions arise mainly near element boundaries while those of even-degree solution arise in interiors [1]. For hyperbolic problems solved by discontinuous Galerkin (DG) methods, we show that errors converge at a higher rate at outflow boundaries than elsewhere [2]. By neglecting errors in superconvergence regions, we are able to construct asymptotically correct spatial discretization error estimates for finite element and DG solutions. We demonstrate their accuracy relative to several compressible flow and MHD problems.

We try to develop unifying principles between these superconvergence phenomena by characterizing solutions in terms of cell Peclet numbers that range from near zero for strongly parabolic problems and tend to infinity for hyperbolic problems. In this way, we seek to construct *a posteriori* error estimates for singularly perturbed parabolic problems.

## References

- [1] S. Adjerid, J.E. Flaherty, and I. Babuška, “A posteriori error estimation for the finite element method-of-lines solution of parabolic problems,” *Math. Models and Meths. in Appl. Sci.*, v. 9, p. 261-286, 1999.
- [2] L. Krivodonova and J.E. Flaherty, “Error estimation for discontinuous Galerkin solutions of multidimensional hyperbolic problems,” *Adv. in Comput. Maths.*, to appear, 2003.