A POSTERIORI ERROR BOUNDS FOR REDUCED-BASIS APPROXIMATION OF PARAMETRIZED NONCOERCIVE AND NONLINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

K. Veroy[a,1], **C. Prud'homme**[a,2], **N.C. Nguyen**[b], and **A.T. Patera**[c]

[a]Department of Mechanical Engineering
Massachusetts Institute of Technology
77 Massachusetts Avenue, Room 3-264
Cambridge, MA 02139
[1]kplveroy@mit.edu, [2]prudhomm@mit.edu

[b]National University of Singapore
Singapore- MIT Alliance
E4-04-10, 4 Engineering Drive 3
Singapore 117576
smap1099@nus.edu.sg

[c]Department of Mechanical Engineering
Massachusetts Institute of Technology
77 Massachusetts Avenue, Room 3-266
Cambridge, MA 02139
patera@mit.edu

We present a technique for the *rapid* and *reliable* prediction of linear-functional outputs of (second-order) elliptic partial differential equations with affine parameter dependence. The essential components are (*i*) rapidly convergent global reduced-basis approximations — (Galerkin) projection onto a space W_N spanned by solutions of the governing partial differential equation at N selected points in parameter space; (*ii*) *a posteriori* error estimation — relaxations of the error-residual equation that provide inexpensive yet sharp bounds for the error in the outputs of interest; and (*iii*) offline/online computational procedures — methods which decouple the generation and projection stages of the approximation process. The operation count for the online stage — in which, given a new parameter value, we calculate the output of interest and associated error bound — depends only on N (typically very small) and the parametric complexity of the problem.

In this paper we develop new *a posteriori* error estimation procedures for noncoercive linear, and certain nonlinear, problems that yield rigorous and sharp error statements for *all* N. We consider five particular examples: the Helmholtz (reduced-wave) equation; a cubically nonlinear Poisson equation; the symmetric eigenvalue problem; Burgers equation; and the Navier Stokes equation. The Helmholtz example, as well as the eigenvalue, Burgers, and Navier Stokes problems, introduce our new lower bound constructions for the requisite inf-sup (singular value) stability factor; the cubic nonlinearity, as well as the Burgers and Navier Stokes problems, exercises symmetry factorization procedures necessary for treatment of high-order Galerkin summations in the (say) residual dual-norm calculation; the Burgers and Navier Stokes (respectively, eigenvalue) problems illustrate our accommodation of potentially (respectively, provably) multiple solution branches in our *a posteriori* error statement. Numerical results are presented that demonstrate the rigor, sharpness, and efficiency of our proposed error bounds, and the application of these bounds to adaptive (optimal) approximation.