CONTINUUM SHAPE SENSITIVITY ANALYSIS OF CRACKS IN FUNCTIONALLY GRADED MATERIALS

B. N. Rao[a], and **S. Rahman**[a,b]

[a]Center for Computer-Aided Design
The University of Iowa
Iowa City, IA 52242-1000
bnrao@ccad.uiowa.edu

[b]Department of Mechanical and Industrial Engineering
The University of Iowa
Iowa City, IA 52242
rahman@engineering.uiowa.edu

In recent years, special type of composite materials known as functionally graded materials (FGMs) have been introduced and applied to the development of structural components that are exposed to non-uniform service requirements. Although the absence of sharp interfaces in FGMs largely reduces material property mismatch, cracks do occur when FGMs are exposed to external loading and environments. The overall mechanical and thermal-mechanical responses and reliability of FGMs are largely determined by the fracture induced by such cracks. While development is ongoing, a number of deterministic methods required to quantify appropriate crack-driving force in FGM have been developed [1]. Probabilistic fracture mechanics (PFM) that blends the theory of fracture mechanics and the probability theory provides a more rational means to describe the actual behavior and reliability of structures. However in PFM, the derivatives of the fracture parameters are often required to predict the probability of fracture initiation and/or instability in cracked structures. The calculation of these derivatives with respect to load and material parameters, which constitutes size-sensitivity analysis, is not unduly difficult. However, the evaluation of response derivatives with respect to crack size is a challenging task, since it requires shape sensitivity analysis [2]. Using a brute-force type finite-difference method to calculate the shape sensitivities is often computationally expensive, in that numerous repetitions of deterministic finite element analysis may be required for a complete reliability analysis. Therefore, an essential need of probabilistic fracture-mechanics is to evaluate the sensitivity of fracture parameters accurately and efficiently.

This paper presents a new method for continuum shape sensitivity analyses of a crack in an isotropic, linear-elastic FGM. The method involves the finite element discretization, the material derivative concept of continuum mechanics, domain integral representation of a *J*-integral or an interaction integral, and direct differentiation. Unlike virtual crack extension techniques, no mesh perturbation is needed in the proposed method to calculate the sensitivity of stress-intensity factors. Since the governing variational equation is differentiated prior to the process of discretization, the resulting sensitivity equations are independent of approximate numerical techniques, such as the finite element method, boundary element method, meshless method, or others. In addition, since the *J*-integral or the interaction integral is represented by domain integration, only the first-order sensitivity of the displacement field is needed. Numerical results show that first-order sensitivities of *J*-integral or stress intensity factors obtained by using the proposed method are in excellent agreement with the reference solutions obtained from finite-difference methods for the structural and crack geometries considered in this study.

References

[1] B. N. Rao, and S. Rahman, "Meshfree Analysis of Cracks in Isotropic Functionally Graded Materials," *Engineering Fracture Mechanics*, v. 70, p. 1-27, 2003.
[2] G. Chen, S. Rahman, and Y. H. Park, "Shape Sensitivity Analysis in Mixed-Mode Fracture Mechanics," *Computational Mechanics*, v. 27, p. 282-291, 2001.