STABILIZED FINITE ELEMENT FORMULATION FOR ADVECTION-DIFFUSION-ABSORPTION PROBLEMS USING FINITE CALCULUS

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Considerable effort has been spent in recent years to derive numerical methods for the solution of the advection-diffusion-absorption equation. Solutions of this type of problems are of the form of real exponential functions. Numerical schemes find difficulties to approximate the sharp gradients appearing in the neighborhood of boundary and internal layers due to high Peclet and/or Damköler numbers.

Stabilized methods to tackle above problems have been based on streamline-upwind-Petrov-Galerkin (SUPG) techniques and subgrid scale (SGS) finite element methods. An extension of the SGS accounting for all the physical regimes of the advection-diffusion-reaction equation has been recently proposed by Hauke [1]. However, as reported in [1] both SUPG and SGS methods (and indeed the standard Galerkin scheme) fail to obtain an stabilized solution for some specific boundary conditions in the exponential typically regime when there is a negative streamwise gradient of the solution.

In this paper a stabilized FEM for the stationary 1D advection-diffusion-absorption equation is presented. The stabilized formulation is based on the modified governing differential equations derived via *finite calculus* (FIC). This technique is based on writing the balance equations over a domain of finite size and retaining higher order terms. These terms incorporate the ingredients for the necessary stabilization of any transient and steady state numerical solution. The FIC differential equations can be used to derive an iterative residual-based scheme for computing the stabilization parameters. The FIC approach has been successfully used to derive stabilized FEM for advective-diffusive problems and incompressible problems in fluid and solid mechanics [2,3].

The FIC formulation here proposed leads naturally to an expression of the stabilization parameter which is a function of the solution and its derivatives. This introduces a non-linearity in the problem and a simple iterative algorithm leading to a stabilized solution in few iterations is presented. It is shown that *the sign of the stabilization parameter* depends on the solution. The correct sign of this parameter is essential to obtain stabilized results for all types of boundary conditions, thus overcoming the problems detected in [1]. The good behaviour of the new stabilized formulation is shown in a number of examples of application.

References

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