EULERIAN VERSUS UPDATED LAGANGIAN ALGORITHMS FOR FILM CASTING

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Two main approaches exist for describing the motion of a body: the material and spatial formulations. Although the different formulations should yield identical results for any smooth motion of a body, for any given type of motion there is often an obvious choice. For instance, fluid mechanics problems lend themselves to spatial, often termed Eulerian (E), formulations. On the other hand, solid mechanics problems are usually best treated using material, or the so-called Lagrangian (L) and updated Lagrangian (UL), descriptions. Although the choice for description of motion is well established for fluid and solid mechanics problems, it is not so clear which choice is best when the material model in question exhibits properties of both a fluid and a solid; that is, when a viscoelastic model is appropriate, as is often the case for polymer processing. The choice of description of motion for polymer processing is also complicated by the common presence of a free surface. This paper builds on a recent study [1] that investigated the tradeoffs between the spatial and material formulations by comparing them for the specific process of film casting.

Film casting is used to produce thin polymer films for such uses as food packaging and magnetic tape. The process involves a molten polymer film extruded from a slot die and then stretched through an air gap by the rotation of a downstream chill roll, which also acts to freeze the polymer. The previous study [1] compared E and UL finite element algorithms for simulating one-dimensional (1D) film casting, for which a closed-form solution exists. The E algorithm was found to be more accurate, faster and simpler to implement. However, the UL algorithm also produced reasonable solutions and it provided a natural and intuitive framework for accommodating viscoelasticity and for investigating the stability of the film. To better understand the differences between the E and UL approaches the current study considers a more complex model that allows for a two-dimensional (2D) geometry.

Unlike 1D film casting, a closed-form solution is not available for 2D film casting. For the 2D case, the discussion focuses on comparing the simulation results of the numerical algorithms. The comparisons shows disagreement, at times considerable, between the geometries predicted for the film. In a general sense, the disagreement in the numerical predictions is a consequence of the highly nonlinear nature of the problem and the strong coupling between the velocity field and the film geometry. In this paper, three potential explanations for differences in predictions are explored: i) the UL algorithm only approximates the E boundary conditions; ii) the solution of the UL algorithm may depend on the initial guess; and, iii) although the E and UL solutions for velocity and geometry are dissimilar, the two solutions are difficult for a finite element algorithm to distinguish between because they are relatively close in an energy sense. The last explanation was found to be the most promising.

References

[1] S. Smith and D. Stolle,"A comparison of updated Lagrangian and Eulerian finite element algorithms for simulating film casting," *Finite Elements in Analysis and Design*,v. 38, n. 5, p. 401-415, 2002.