A MATHEMATICA-SUPPORTED STUDY OF OPTIMAL MEMBRANE TRIANGLES WITH DRILLING FREEDOMS

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This paper compares derivation methods for constructing high-performance membrane triangles with corner drilling freedoms. The ultimate objective is to construct optimal elements of this configuration. The term “optimal” is used in the sense of exact inplane pure-bending response of rectangular mesh units of arbitrary aspect ratio. Following a comparative summary of element formulation approaches, the construction of an optimal 3-node triangle using the ANDES formulation is presented. The construction is based upon techniques developed by 1991 in student term projects, but taking advantage of the more general framework of templates developed since. Effective use of this framework has been made possible by the availability of advanced symbolic computation programs on personal computers.

The optimal element that fits the ANDES template is shown to be unique if energy orthogonality constraints are enforced *a priori*. Two other formulations are examined and compared with the optimal model. Retrofitting the conventional LST (Linear Strain Triangle) element by midpoint-migrating by congruential transformations is shown to be unable to produce an optimal element while rank deficiency is inevitable. Use of the quadratic strain field of the 1988 Allman triangle, or linear filtered versions thereof, is also unable to reproduce the optimal element. Moreover these elements exhibit serious aspect ratio lock. These predictions are verified on numerical benchmark examples.