CONTROL OF POLLUTANT CONCENTRATION IN SHALLOW WATER FLOW USING OPTIMAL CONTROL THEORY

Jinya Kobayashi[a] and Mutsuto Kawahara[b]

Department of Civil Engineering
Chuo University
Bunkyo-ku, Tokyo-to, Japan
[a]jin526@kc.chuo-u.ac.jp
[b]kawa@civil.chuo-u.ac.jp

This paper presents a study of the control of pollutant concentration in the shallow water flow using the optimal control theory. In Japan, rapid the development of social economy and the urbanization have been making the pollution expand. In recent years, inflow of the pollutant from a river to the sea has increased and accumulated as the economy of coast area has grown. It leads to a lot of problem. For example, to give off a bad smell, the generation of red tides, the eutrophication of water quality, and so on. The water pollution problem has been serious. Therefore, it is necessary to know the optimal inflow for controlling the pollutant concentration to the target concentration. In this study, to analyze the control of pollutant concentration, the optimal control theory is applied. The optimal control theory is a methodology that calculates the optimal controlled variable for changing into the target phenomenon to the aiming state. Therefore, the purpose of this study is to obtain the optimal controlled variable so as to minimize the pollutant concentration using the optimal control theory and finite element method.

The non-linear shallow water equation is used for the analysis of flow behavior, and the advection-diffusion equation is used for the analysis of pollutant concentration. The quasi-linear approximation of advection velocity is given by the Adams-Bashforth formula that has the second order accuracy. To solve the basic equations and the adjoint equations, The Crank-Nicolson method is applied to the temporal discretization. This method is capable of taking the large time increment and superior in stability. The finite element method that bases on the bubble element function is applied to the spatial discretization. The Sakawa-Shindo method is employed as a minimization technique.

As a result of this research, it turns out that the pollutant concentration in the objective points could be lowered and the optimal control is performed.

References

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