SOLUTION METHODS FOR DIRECT AND INVERSE PROBLEMS OF FRACTIONAL DISPERSION

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The problem of radio-nuclide transport in highly non-homogeneous unsaturated porous medium with fractures becomes important when the probabilistic safety assessment of the long-life nuclear waste storage is concerned. The main challenge here is associated with a multitude of scales of the process. This makes the conventional numerical models difficult to use and often leads to significant differences between the field experiment data and numerical predictions.

A very natural assumption to be made in the modeling of transport processes in highly non-homogeneous medium is to assume their linearity. Non-local linear models result in integral equations such as

$$\varphi(\vec{r},t)=\int_{t_0}^{t}\left\{\iiint\left[G(\vec{r}-\vec{r}',t-t',\alpha,\beta,\gamma...)\cdot\varphi(\vec{r}',t')\right]\cdot d\vec{r}'\right\}\cdot dt' \qquad (0.1)$$

where φ is the concentration of nuclides, $\alpha,\beta,\gamma...$ are parameters of Green function G, which define the properties of the surrounding medium.

In the present work, two different types of nuclei are considered in a phenomenological manner and the integral problem (0.1) is reduced to integral-differential equations with fractional derivatives. Although analytical properties of such solutions are well known [1-4] the theory for their numerical solution is less developed. New efficient numerical methods for equations with fractional space and time derivatives have been developed in one and multiple dimensions.

The main task for the linear phenomenological theory of transport in highly non-homogeneous medium is seen in determining the unknown parameters α,β,γ from the field measurement data, or in solving the inverse problem. We propose to make use of artificial neural networks to solve the inverse problem. A self-learning method of the neural network has been developed basing on the data from the results of direct problem solution.

References

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