EFFICIENT LOCAL BOUND-PRESERVING CONSERVATIVE INTERPOLATION

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The goal of this presentation is to present a new efficient, conservative, local-bound-preserving, second-order accurate algorithm for remapping (conservative interpolation) of a cell-centered discrete function between two close grids, that does not require finding intersections between the cells of the two grids.
The statement of the conservative interpolation can be described as follows. We are given mean values of some unknown underlying function (which we will call density) on some grid (old grid). These discrete values define total mass, which is just the sum of these values multiplied by the corresponding volumes over the entire mesh. The goal of remapping is to find an accurate approximation for the means over the new mesh (which is close to the old mesh), such that total mass is conserved.
The algorithm consists of three stages. In the first stage we construct a discontinuous piecewise linear reconstruction based on values of the discrete function on the old grid. It involves some estimates for the gradient, however we do not use any limiters.
In the second stage, the formulas for new masses are written in flux form, that is, the mass of each new cell will be set to the mass of the corresponding old cell plus terms which define exchange of mass with its nearest neighbors (fluxes). Using the flux form trivially guarantees conservation of total mass. We use a quadrature to compute these fluxes, which does not require finding the intersections of new and old cells. This procedure uses the notion of swept regions (regions swept by the motion of the cell faces).
It may happen that the mean densities in the new cells will be out of local bounds, for example, they can be negative even if the original means were all positive. The cause of this can be: either use of unlimited gradients in the reconstruction, approximate integration, or both. To fix out of bound mean densities we introduce a third stage - repair.
In this stage masses of new cells are redistributed in such a way that the modified new means are in the range of local bounds, and they are changed as little as possible compared with those obtained as a result of the second stage.