MULTI-LEVEL BOUNDARY ELEMENT METHODS FOR STOKES FLOWS

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An extremely robust, fast, and accurate multi-level boundary element method (MLBEM) has been developed for the solution of two-dimensional Stokes equations. The numerical method represents an extension of the multi-level multi-integration (MLMI) approach, originally proposed by Brandt and Lubrecht [1], and most recently extended by Grigoriev and Dargush to the solutions of general boundary value problems arising in steady-state heat diffusion [2] and acoustics [3]. Here, the MLMI algorithm is embedded into a bi-conjugate gradient method to allow very efficient evaluation of the matrix-vector and matrix-transpose-vector multiplications. Additionally, the use of the multi-grid technique [4] allows us to obtain converged numerical solutions in just a couple of multi-grid cycles independent of the boundary element discretization.

The result of this development is a fast MLBEM algorithm that allows to achieve nearly $O(N \ln N)$ operation counts, whereas the conventional boundary element formulations require $O(N^{5/2})$ and $O(N^3)$ operations when using iterative and direct solvers, respectively. Note that the memory requirements for the conventional boundary element methods are of order $O(N^2)$ since the global matrix is dense due to the Stokeslets. Meanwhile, the fast multi-level formulation reduces the memory requirements also to the order $O(N \ln N)$.

In order to demonstrate the attractiveness of the MLBEM algorithm for the Stokes flows, we consider an example problem with an analytical solution and perform a detailed investigation of the CPU and memory requirements while maintaining the desired level of solution accuracy. We demonstrate exceptional performance of the fast boundary element formulation for the problem considered in this study. In particular, the CPU times and memory requirements for the MLBEM algorithm can be orders of magnitude less than those for the conventional techniques with essentially no loss in accuracy.

References

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