A HIGH-ORDER SPACE-TIME COUPLED LEAST-SQUARES FINITE ELEMENT FORMULATION FOR INCOMPRESSIBLE FLUID FLOWS

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We present a finite element formulation based on least-squares variational principles for the numerical solution of the stationary and non-stationary Navier-Stokes equations governing viscous incompressible fluid flows. The Navier-Stokes equations are expressed as an equivalent set of first-order equations by introducing the vorticity or the velocity gradients as independent variables. The use of least-squares principles leads to a variational unconstrained minimization problem where the approximation spaces for velocity and pressure can be chosen independently, i.e. stability requirements such as the inf-sup condition never arise. For the non-stationary case, we present a formulation where the effects of space-time are coupled. This results in a true space-time least-squares minimization procedure, as opposed to a space-time decoupled formulation where a least-squares minimization procedure is performed in space at each time step.

In the context of least-squares finite element models for the incompressible Navier-Stokes, predominantly low order nodal expansions have been used. Although not commonly emphasized, low order nodal expansions tend to lock and reduced integration techniques must be used to obtain acceptable numerical results. When enough redundant degrees of freedom are constrained the least-squares finite element solution using reduced integration yields a collocation finite element solution. However, the collocation solution may not always be reliable and the least-squares functional cannot be used to measure the quality of the solution. As shown and concluded in our previous work [1,2], use of high order nodal/modal expansions and full integration are the appropriate way to truly minimize the least-squares functional. The quality of the numerical solution may be judged by the value of the least-squares functional, which decays exponentially fast as the expansion order of the nodal/modal basis is increased [1,2].

High-order nodal or modal expansions in space-time are used to construct the discrete finite element model. Exponentially fast decay (spectral convergence) of the least-squares functional, constructed using the L_2 norms of the equations residuals, is verified for smooth solutions of the stationary and non-stationary Navier-Stokes equations. The space-time coupled formulation has no time step stability restrictions and is spectrally accurate in both space and time. The use of least-squares variational principles leads to a symmetric and positive-definite system of algebraic equations. The system of equations thus obtained is linearized by Newton's method and solved by the preconditioned conjugate gradient method in matrix-free form. For the stationary case, numerical results are presented for flow over a backward-facing step and three-dimensional lid-driven cavity flow. For the non-stationary case, numerical results are presented for impulsively started lid-driven cavity flow and flow around a circular cylinder.

References

[1] J. P. Pontaza and J. N. Reddy "Spectral/*hp* least-squares finite element formulation for the Navier-Stokes equations," *Journal of Computational Physics*, submitted.
[2] J. P. Pontaza and J. N. Reddy "Space-Time coupled spectral/*hp* least-squares finite element formulation for the incompressible Navier-Stokes equations," *Journal of Computational Physics*, submitted.