GRID-BASED POISSON SOLVER WITH BOUNDARY CONDITIONS OBTAINED USING THE FAST MULTIPOLE METHOD: AN OPTIMAL COMBINATION FOR VORTEX-IN-CELL METHODS

G. Daeninck, R. Cocle and G. Winckelmans

Université catholique de Louvain (UCL), Mechanical Engineering Departement
Bât. Stévin, Place du Levant 2, B-1348 Louvain-la-Neuve, Belgium
daeninck@term.ucl.ac.be and winckelmans@term.ucl.ac.be

In vortex methods, an efficient alternative to fast multipole methods is to use grid-based Poisson solvers. Efficient solvers have an $\mathcal{O}(M \log M)$ computational cost (M being the number of grid points), and the constant in front is much smaller than that associated with the best $\mathcal{O}(N \log N)$ multipole methods.

This approach then calls for a hybrid *particle-grid* method: the so-called *vortex-in-cell* (VIC) method. At each time step, the method requires the projection of the particle strengths onto the grid, and the projection of the grid-solved velocity (and its gradient in 3D) back onto the particles. These are easily achieved using high order interpolation/redistribution schemes (such as Λ_3 or M_4').

As $\nabla^2 \boldsymbol{\psi} = -\boldsymbol{\omega}$ is solved on a grid, one must either assume periodic problems, or one must provide proper boundary conditions on the sides. For external flows, one would use a grid extending sufficiently far that an analytical (still approximate) boundary condition can be used, with as little effect as possible in the flow region of interest. The better the quality of the boundary condition, the larger the required grid, and thus the larger the part which is "wasted" (as it serves to compute a field that is not used by the vortex method). This is a weak point of the VIC approach.

We here propose an original approach where the "exact" boundary condition to be supplied to the grid solver is computed using the fast multipole method. This combination is quite optimal: a compact VIC grid, enclosing tightly the non-zero vorticity field, can be used while the boundary condition is still enforced properly and efficiently.

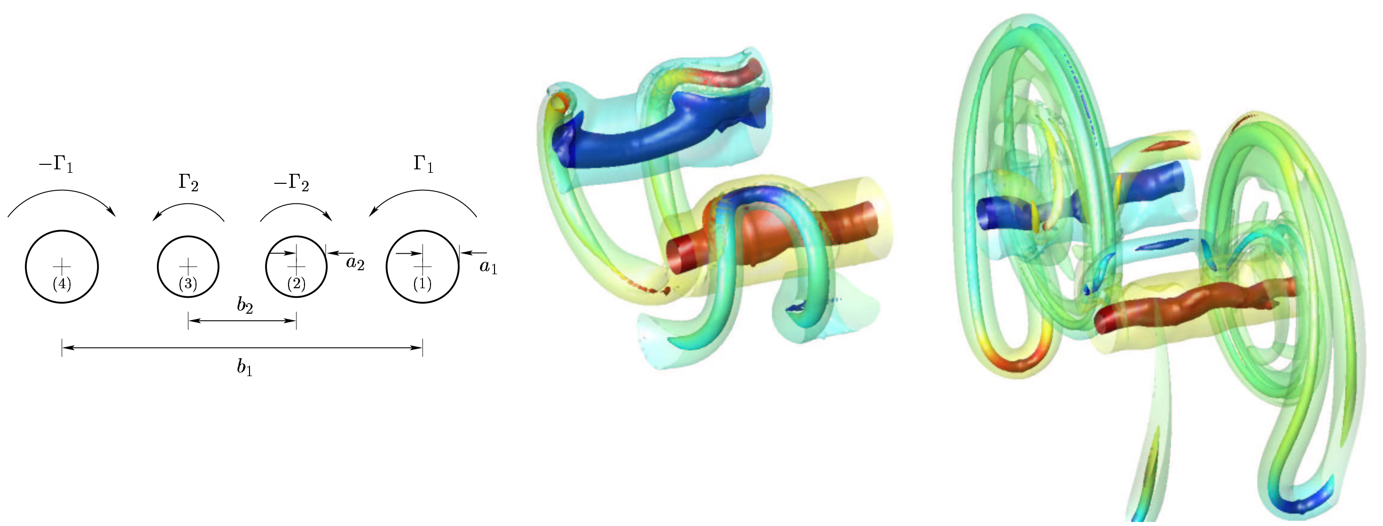

Figure 1: *Instability of a four-vortex system computed using the combined "VIC + fast multipole" method. Initially, $\Gamma_2/\Gamma_1 = -0.3$, $b_2/b_1 = 0.3$ (spacing ratio), $a_1/b_1 = 0.075$ and $a_2/a_1 = 2/3$, each vortex having $\omega(r) = \frac{\Gamma}{\pi}\frac{a^2}{(r^2+a^2)^2}$. Times shown are $t\,\Gamma_1/b_1 = 12$ and 16. The problem is periodic in x (with $L_x/b_1 = 0.983$, from most unstable mode) but not in y and z; an x-periodic version of the multipole method is used to obtain the boundary conditions on the VIC grid. The Reynolds number is $\Gamma_1/\nu = 5000$ and the discretization is $h/L_x = 64$. The VIC grid grows from $64 \times 124 \times 60$ to $64 \times 248 \times 170$ (at $t\Gamma_1/b_1 = 20$), and the number of particles grows from 364800 to 650350. The simulation time is 17 hours on a single processor Pentium 4 (2.4 Ghz) with 1 GB of memory.*