PARALLEL TECHNIQUE FOR NAVIER-STOKES EQUATIONS BASED ON THE ITERATIVE SPACE-MARCHING METHOD

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The iterative space-marching method (IMM) for incompressible and compressible Navier-Stokes equations is developed in works [1-4] and other. This method is applied to solve steady and unsteady, internal and external 1D, 2D, 3D problems. The main characteristic feature of the method is the use a common procedure for solving various types of problems, namely: marching on one coordinate for 1D, 2D problems and on two coordinates for 3D problems. Thus a single algebraic procedure is used (repeatedly) for all problems. This is the procedure of finding the vector of unknown grid functions along transverse line. It is analytically proved that stability and convergence of the method are unconditional for incompressible problems [1, 2]. Good stability and convergence of the method for solving compressible flows are demonstrated on numerical examples [2, 3].

In this work we present computational schemes (in IMM framework) suitable for solving steady and unsteady problems with parallel algorithm. We prove that they have unconditional convergence. The proofs are given for incompressible fluid schemes that are special cases of compressible ones at zero Mach number. Based on these schemes one processor may be used for computations of the previous algebraic procedure in each marching station on any time layer or global iteration.

First consider the scheme for steady problems. The modified scheme of IMM is efficient for these problems. This scheme is based on the principle of convergence in time to a steady state. According to this scheme each time-step is realized with one space marching sweep. In order to formulate a scheme that is realized with a parallel algorithm we change finite-difference approximations of derivatives from velocity components with respect to marching coordinates. Then we prove that this scheme converges in time unconditionally for 2D and 3D problems.

Unsteady problems are solved using the general scheme of the IMM [1, 2]. In this case the solution at each temporal layer is found using global iterations (GI). Each GI is a space-marching sweep. We make similar changes in the general scheme to the changes we made above for the modified scheme. Then we prove that this scheme converges unconditionally in GI for 2D and 3D problems.

The facts of convergence are confirmed by solving test problems using conservative form of equations. Estimation of speed-up is given. Parallel schemes for cases of approximations with second order of marching derivatives [2] and for compressible Navier-Stokes equations [3] may be formulated (in IMM framework) in a similar way. This work is summarized in [5].

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References

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