HYPERSINGULAR BOUNDARY SURFACE FINITE ELEMENT METHOD

A.R. Hadjesfandiari

Department of Civil Engineering
State University of New York at Buffalo
Amherst, NY 14260
ah@eng.buffalo.edu

The theory of fundamental boundary eigensolutions presented in [1-5] shows that the solution to every field such as potential or elastostatics can be written as a linear combination of some boundary orthogonal eigenfunctions. It has also been shown how to modify the traditional finite element methods with nodal sources (forces) to nodal fluxes (tractions) in scalar (vectorial) boundary value problems. This makes the traditional finite element method consistent with the theory of fundamental boundary eigensolutions. It is seen that this modified finite element method is simply an indirect general discretized Fourier analysis.

The finite element formulation traditionally is derived from virtual work or a weak formulation dealing with domain integrals. In this presentation, we formulate the finite element method based on a weak formulation in the form of a double boundary integral. The kernel in this integral equation is hypersingular which requires interpretation as Hadamard finite part. The boundary stiffness matrix derived from this method is more accurate than the condensed boundary stiffness matrix derived from the traditional method.

In this presentation, the results for the potential and elastostatic problems are compared with those from the traditional finite element approach. This shows that increasing the number of internal nodes in the traditional finite element model improves the boundary stiffness matrix toward the boundary stiffness matrix from the hypersingular boundary integral approach.

Additionally, this new formulation gives us a better insight into the finite element method to interpret systematically the numerical results from even the traditional method. By bringing the theory of distributions to study divergent series related to hypersingular kernels, we can advance further, toward non-classical boundary value problems such as fractal field problems.

References

[1] Hadjesfandiari A. R. Theoretical and computational concepts in engineering mechanics. Ph.D. dissertation, State University of New York at Buffalo, 1998.
[2] Hadjesfandiari, A. R. and Dargush, G. F., 'Theory of boundary eigensolutions in engineering mechanics', *J. Appl. Mech.*, ASME, 68 (1), 101-108 (2001).
[3] Hadjesfandiari, A. R. and Dargush, G. F., 'Computational mechanics based on the theory of boundary eigensolutions', *Int. J. Num. Meth. Engrg.*, 50, 325-346 (2001).
[4] Hadjesfandiari, A. R. and Dargush, G. F., 'Boundary eigensolutions in elasticity, Part I. Theoretical development', *Int. J. Solids Struct.*, 38, 6589-6625 (2001).
[5] Hadjesfandiari, A. R. and Dargush, G. F., 'Boundary eigensolutions in elasticity, Part II. Application to computational mechanics', *Int. J. Solids Struct.*, 40, 1001-1031 (2003).