NOVEL METHODS FOR REDUCING THE DIMENSIONS OF LARGE-SCALE PROBLEMS: APPLICATIONS TO FLUID MECHANICS

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Of fundamental importance in any numerical computation is the optimal, low dimensional representation of an essentially infinite dimensional mechanics problem. Such a representation is important in understanding the underlying physics of the problem, building models of the system to various levels of accuracy, reducing simulation time and memory requirements, etc. Some of the existing methods in the area of mechanics are singular perturbation technique and Karhunen-Love decomposition.

In this paper, we introduce a novel technique for getting simple models of unsteady fluid phenomena. This method is quite general and can be applied to other mechanics problems with ease. The main idea behind these methods is deleting the weakly controllable and weakly observable states of the system after the controllability and the observability gramians of the system are aligned through a similarity transformation. The relative importance of a state in the input-output behavior of the system is given by the corresponding Hankel singular value. We will show that for Stream-wise constant Navier-Stokes equations linearized about Couette flow the Hankel singular values drop very steeply. The error (H_∞ norm of the difference of original and truncated fluid transfer functions) in the projection is given by twice the sum of the Hankel singular values of the truncated states. Numerical computations done on Couette flow using spectral method, Fourier in span wise direction and Chebechev collocation in wall normal direction will be presented.