WELL-POSEDNESS OF AN ELASTIC INVERSE PROBLEM ARISING IN MEDICAL IMAGING

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Elastography, the imaging of soft tissue on the basis of (shear) elastic modulus, is an emerging diagnostic imaging method. The technique relies on being able to image soft tissue while it is being deformed by a set of externally applied forces. Through image processing, the displacement (or sometimes velocity) field everywhere in the region of interest is measured. An inverse problem for the elastic modulus results, given the measured displacement fields, an assumed form of the tissue's constitutive equation (e.g. linear elastic), and the law of conservation of momentum. We formulate, study and solve this inverse problem. We find that the standard elastography inverse problem is nonunique. We describe new (and practical) formulations that are unique, discuss continuity of the solution on the data, and existence of solution. Further, we introduce a novel stable numerical method required to solve the resulting advective hyperbolic systems of equations, and present examples.

References

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