DETERMINING ELASTIC AND ACOUSTIC PARAMETERS FROM TRAVEL TIMES

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The inverse kinematic problem can be described as follows An acoustic object Ω in 3D is probed for the wave speed $c(x)$, $x = (x_1, x_2, x_3) \in \Omega$, by measuring travel times (times-of-flight) between points on the boundary Γ of Ω. A travel time function $T(x, y)$ with source in $y \in \Gamma$ is related to the wave speed via the eikonal equation $c(x)|\nabla_x T(x, y)| = 1$. One asks to recover c from measurements $T(x, y)$ made for all $x, y \in \Gamma$. Travel time inversion is an essential part of ultrasound techniques in medicine and mechanics (non-destructive evaluation).

From a geometrical point of view T measures the distance between two points with respect to the isotropic Riemannian metric $ds^2 = c(x)^{-2}dx^2$. Therefore $d(x, y) = T(x, y)$ for $x, y \in \Gamma$ is also called the boundary distance function of the metric. One asks if the boundary distance function uniquely determines the metric and therefore also c. The case considered above corresponds to an isotropic medium. In several physical examples the index of refraction is anisotropic, i.e. the wave speeds depend on direction, arises in several physical situations. One example is when an elastic medium has residual stress. In an anisotropic medium we model the wave speed as given by a symmetric, positive definite matrix $g = (g_{ij})(x)$, that is, a Riemannian metric in mathematical terms. The problem is to determine the Riemannian metric from the lengths of geodesics (ray paths) joining points in the boundary which we will denote now by $d_g(x, y)$, to make clear the dependence on the metric g.

In this talk we will describe recent progress on the problem of determining g from d_g. In particular we will outline the result of the speaker and L. Pestov [1] that in the two dimensional case one can recover the metric g under certain natural conditions and up to the natural obstruction. The method also gives a simple reconstruction method for the isotropic sound speed in 2D from the travel times. In 3D a result of the speaker and P. Stefanov [2] shows that this is the case if the medium is slightly anisotropic.

References

[1] L. Pestov and G. Uhlmann, "Two simple dimensional simple compact two dimensional manifolds are boundary distance rigid", preprint.

[2]P. Stefanov and G. Uhlmann, "Microlocal approach to boundary rigidity", preprint.