ELASTODYNAMIC IDENTIFICATION OF SUBTERRANEAN OBJECTS VIA THE LINEAR SAMPLING METHOD

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The inverse scattering problem of identifying elastic obstacles hidden in a semi-infinite solid from surface seismic measurements is investigated within the framework of the linear sampling method. Over the past decade, the latter approach has been the subject of mounting attention in inverse acoustic and electromagnetic theories [1,2,3] dealing with far-field wave patterns. So far, however, its application to the interpretation of near-field elastic waveforms, which are of great importance in engineering seismology and defense applications, has eluded an in-depth scrutiny. Aimed at bridging this gap, a three-dimensional inverse analysis of elastic waves scattered by a subterranean obstacle (or a system thereof), manifest in the surface ground motion patterns, is formulated within the framework of the linear sampling method. In the context of non-invasive seismic sensing of obstacles hidden in a semi-infinite solid: $\Omega = \{(z_1, z_2, z_3)|z_3 > 0\}$, the linear sampling method is shown to revolve around the integral equation

$$\int_S u_i^{obs}(\boldsymbol{x}^q; \boldsymbol{\xi}, \omega)\, g(\boldsymbol{\xi}; \boldsymbol{z})\, dS_\xi = \hat{u}_i^3(\boldsymbol{x}^q, \boldsymbol{z}; \omega), \qquad \boldsymbol{x}^q \in S, \qquad \boldsymbol{z} \in \Omega \tag{1}$$

where S is the measurement surface; $u_i^{obs}(\boldsymbol{x}^q; \boldsymbol{\xi}, \omega)$ are the experimental observations of ground motion at sensor locations $\boldsymbol{x}^q$ due to a surface point source acting normally at $\boldsymbol{\xi}$; $\hat{u}_i^3$ is the elastodynamic half-space Green's function, and $\boldsymbol{z}$ is a fixed trial, i.e. sampling point. Theoretical investigations have indicated that the norm of the solution to (1), $||g||$, becomes unbounded as the sampling point $\boldsymbol{z}$ approaches the surface of the hidden scatterer from its interior.

For computational purposes (assuming a sufficient density of source locations), kernel u_i^{obs} can be suitably interpolated over S, and (1) can be converted to a linear algebraic system: $\boldsymbol{A}\,\boldsymbol{g}(\boldsymbol{z}) = \hat{\boldsymbol{u}}(\boldsymbol{z})$ where $\boldsymbol{A}$ is a *constant* matrix whose properties are dictated by the experimental observations u_i^{obs}. In what follows, a stable solution $g(\cdot; \boldsymbol{z})$ of the ill-posed equation (1) can be obtained by minimizing the Tikhonov functional

$$\mathcal{J}(\boldsymbol{g}) := \|\boldsymbol{A}\boldsymbol{g}(\boldsymbol{z}) - \hat{\boldsymbol{u}}(\boldsymbol{z})\|^2 + \alpha\|\boldsymbol{g}(\boldsymbol{z})\|^2 \to \min \tag{2}$$

where the regularization parameter α is chosen a posteriori by the dicrepancy principle of Morozov. On the basis of (2), $||\boldsymbol{g}(\boldsymbol{z})||$ is established as a computationally-efficient probing tool that, through the regions of its bounded values, effectively indicates the location and shape of subterranean obstacles. To this end, the mathematical basis of (1) and computational features of (2) are presented for the case of near-field, 3D elastodynamics. A set of numerical examples with synthetic measurements is included to illustrate the performance of the method.

References

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