

ON THE EVALUATION OF CONFIGURATIONAL FORCES WITH ENRICHED FINITE ELEMENT APPROXIMATIONS USING DOMAIN INTEGRALS

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We present the results of recent multi-dimensional simulations based on a sharp interface theory describing the chemo-mechanical response of synthetic hydrogels [1]. The primary unknowns in the theory are the deformation of the network and a chemical potential that accounts for the energy change associated with solute transport. In addition to the standard bulk and interfacial equations imposing force balance and solute balance, the theory involves an ancillary interfacial equation imposing configurational force balance. In this work, we focus on a specialization of the theory in which the only coupling between the deformation and potential fields is through the normal configurational balance on the interface. We present a hybrid eXtended-Finite-Element/Level-Set Method (XFE/LSM) [2] for obtaining approximate solutions to the evolution equations arising under this specialization.

The fidelity of our approximate solutions is shown to rest upon the accurate evaluation of the interfacial jumps of the solute flux and the bulk configurational traction. To perform these evaluations with enriched finite element approximations, we generalize the domain integral method for the associated balance laws. In the special case when the interface aligns with the mesh and the nodal shape functions are used as weighting functions, this approach yields discrete material node point forces, akin to those arising in the work of Steinmann et al. [3] and Mueller et al. [4]. Using several numerical examples, we show how the use of domain integrals results in a substantial increase in accuracy in comparison to standard post-processing procedures. Finally, simulations of hydrogel swelling are provided to illustrate the robustness of the new techniques and the efficacy of the XFE/LSM.

References

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