Material forces in computational mechanics : Variational ALE method

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In the finite element analysis, numerical solution for nodal variables is sought over a fixed mesh by assuming that the governing functional, e.g. strain energy in elasticity, is dependent only on nodal variable [1]. However, numerical solution is also dependent on mesh, which is the motivation for mesh adaption. For the case of inhomogeneous system the numerical solution is also dependent on material configuration [2]. Consideration of material forces, by considering the dependence of the governing functional on the nodal coordinates, provides elegant framework to account for such a dependency, since the material forces are the thermodynamic forces associated with the nodal coordinates. This enables efficient solution for optimal mesh and material configuration. To this end, we propose Variational ALE method within the material force framework.

In this method in addition to solution for nodal variables, mesh itself is obtained as the direct solution of the problem. In case of the inhomogeneous system material configuration is also obtained as the direct solution. The solution so obtained satisfies material force balance and hence provides precise criterion for mesh adaption. The resulting mesh adaption scheme is devoid of error estimates and interpolation errors associated with mesh-to-mesh transfer. In case of dynamics, VALE provides variational integrator with horizontal variations, which is symplectic and momentum-preserving with good long-time energy behavior.

It is interesting to note that, in the context of discrete Noether's theorem, presence of nodes breaks translational symmetry of the governing functional, which is similar to the defects in the continuous case. Hence, nodes can be considered as discrete defects. This can also be explained from the fact that, in a mesh there is always some neighborhood in which the nodes are singularities, which is not true in the continuous case. Hence the optimal mesh is the mesh corresponding to the case with vanishing nodal (discrete defects) material forces.

Salient features of this framework have been demonstrated with several examples. This method has been used to obtain optimal mesh in static and dynamic situations. In static case in addition to mesh adaption, J-integral has been evaluated accurately, without additional calculations, such as path or domain integral evaluation. In the dynamic case, it has been used as a shock capturing scheme in case of elastic rod. Further, it has been used to solve for optimal material configuration to study the symmetry breaking transitions in equilibrium shapes of coherent precipitates.

References

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