Minimal C^n Conforming Meshfree-FEM Hermite Hierarchy

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Abstract

In this work, a minimal, smooth ($C^n, n \geq 1$), globally compatible Hermite interpolation hierarchy is constructed (discovered) in the framework of reproducing kernel element method (RKEM) for arbitrary multiple dimensional domains.

The so-called reproducing kernel element method (RKEM) is a hybrid meshfree and finite element method, which combines the virtues of both methods, that are: (1) highly smoothness of the interpolation, (2) Kronecker delta property, and (3) non-local interpolation property.

By utilizing these advantages, we have successfully constructed a first interpolation hierarchical structure that has been both minimal degrees of freedom and higher order smoothness and continuity over a multi-dimension discretization. This special construction enables a hierarchical structure of generalized Kronecker property, i.e. $\Psi_I^{(n)}(\mathbf{x}_J) = \delta_{IJ}, n = 0, 1, 2, \cdots$. This work is the latest breakthrough of a half century old outstanding problem — construction of a minimal, higher order continuous $C^n(\Omega), n \geq 1$ finite element interpolation field on an arbitrary mesh or subdivision of multiple dimension.

The essence of the method is to hybrid two partition of unities: a subdivision partition of unity and a special meshfree partition of unity. By carefully distributing meshfree particles, we have shown that one can construct any order continuous interpolation fields on general triangular or tetrahedron finite element mesh in multiple dimensions. In the new method, a notion of *partition polynomial* is first conceived, which is a continuous extension of compact supported finite element shape function. The globally defined partition polynomials are $C^\infty(\mathbb{R}^d)$ functions, and they have some unique properties. For instance, partition polynomials defined in a patch (element) form a unity. By combining a *patch unity* (a subset of partition polynomials) together with compactly supported meshfree interpolant, we can form a global basis functions that can be made as smooth as desired, and at the same time they can interpolate at arbitrary order in multiple dimensions.

The newly invented meshfree-FEM Hermite hierarchy could have a significant impact on simulations and modelings of thin shell structures, applications of complementary variational principles, and simulations and modelings of materials with strong gradient effects.

Examples of compatible Hermite RKEM hierarchical interpolations are illustrated, and they are tested in the Galerkin procedure to solve differential equations.

References

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