APPROXIMATION BY PARTICLE SHAPE FUNCTIONS ASSOCIATED WITH GENERAL (NON-UNIFORMLY DISTRIBUTED) PARTICLES IN R^n

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Approximation results are presented that apply to the shape functions used in a wide variety of particle methods. Classically, approximation results for the case of uniformly distributed particles were first proved using the Fourier Transform. Our first result is a new proof in the case of uniformly distributed particles that doesn't use the Fourier Transform. Our second result generalizes this proof to cover general (non-uniformly distributed) particles. This (second) result can be applied to the shape function employed in a wide variety of particle methods, establishing approximability results for these shape functions. These results apply to the situation in which one shape function is associated with each particle. We will also present results for the situation in which several shape functions are associated with each particle. Using these results we can establish the rate of convergence for meshfree methods that employ, e.g., least-squares particle shape functions or reproducing kernel particle shape functions. This settles an important open question for general (non-uniformly distributed) particles.