A COMPARISON OF LOCAL AND GLOBAL FORMULATIONS OF THE MESH OPTIMIZATION PROBLEM USING THE FEASIBLE NEWTON SOLUTION METHOD

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A significant body of research has recently been developed that uses optimization-based approaches to move the vertices of a mesh to improve its quality. These techniques can be classified into two broad categories based on the problem formulation. Local formulations consider a single vertex at a time. An optimization problem based on that vertex and its adjacent vertices is solved and some number of sweeps over the the mesh is performed to improve the overall quality. Each optimization problem is small and is typically easy to solve. The computational cost can be controlled by performing a small number of sweeps over a mesh, but it is difficult to determine if an optimal solution has been obtained. In constrast, global techniques solve the mesh optimization problem by relocating all of the vertices simultaneously. These optimization problems are characterized by a large number of design variables and can be difficult to solve numerically [1]. Computational costs can be controlled by terminating the solver before convergence is obtained if the method converges to the solution monotically. There are a number of tradeoffs between the two approaches and no systematic study has been performed to answer questions pertaining to which of the two methods is most likely to find the optimal solution and which is the most cost effective.

We present an algorithmic framework that can be used to provide a consistent basis of comparison between the local and global formulations. In both cases, we use the mean ratio quality metric, and an objective function based on an ℓ_2 objective function which targets improvement in the average element quality. The resulting optimization problems are solved using a Feasible Newton method [1]. This technique solves a quadratic approximation of the objective function to find a direction and performs a linesearch along this direction to find an improved point. In previous tests that compared different methods for solving the global formulation, we found this method to be the most robust, fastest, and most scalable. In addition, it converges monotonically to the solution, and can therefore be used to test the efficacy of early termination in a global formulation.

By analyzing a number of different test cases, we identify the conditions for which one formulation is preferred over the other. In particular, we present results for simplicial meshes in two- and three- dimensions in which the initial mesh quality and problem size vary dramatically. Preliminary results indicate that a global-based formulation has many advantages for the mesh optimization problem in terms of time to find the locally optimal solution and the improvement achieved for a fixed computational cost.

References

[1] L. Freitag, P. Knupp, T. Munson, S. Shontz, "A comparison of optimization software for mesh shape-quality improvement problems." *Proceedings of the 11th International Meshing Roundtable* , p. 29-40, 2002.