Hybrid Mesh Generation Based on Metric Identity in Discrete Form

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Our objective is to generate hybrid meshes by extruding prisms from a triangular mesh on the body surface into the field. The mesh generating equations in difference form are derived from the metric identity, which simply states that the sum over all face vectors normal to a volume element must vanish.

In a previous paper, starting from the metric identity in its differential conservative form the well known Poisson equations for structured grid generation were derived including a precise analytical definition of all 9 control functions [1]. This was achieved by just decomposing the face normal vectors into the grid line vectors being connected to the unkown central point. The so formulated identity becomes a grid generation equation, once the control functions are specified. The most simple specification, setting all control functions to zero, results in the face weighted Laplacian equation with favourable smoothing properties. The metric identity can thus be conceived as an expansion of the Laplacian equation about the central point.

Since for unstructured meshes there is no underlying computational space we construct a discrete model of finite volume elements around a central point and consider 3 neighbouring level faces being connected by triangular prisms. For each such prism we introduce 3 line vectors - 2 within a level face, one in marching direction -, which allow us to define the face vectors normal to each prism. Proceeding further as in the structured continuous case we cast the metric identity into a sum of terms, in which the line vectors are weighted by coefficients consisting of volumes and of scalar products of face vectors. Since the line vectors are partly connected to the central point, specifying the coefficients we obtain an elliptic grid generation equation in finite difference form.

For smoothing purposes the Laplacian coefficients of the line vectors in lateral directions are elaborated in finite difference form from two-dimensional considerations. With the so obtained coefficients the Laplacian solution for a structured cell made up of a triangulated quad or hexagon is confirmed. The building law of the coefficients is simply assumed to also hold on any combination of triangles. The coefficients specified for smoothing or control purposes are superimposed with direction dependent weights.

In order to march the solution off the body surface the elliptic difference equation is embedded in a parabolic procedure. From the lateral mesh on the first or lowest level grid points on the second and third level are algebraically forecast, the solution is iterated on the inner second level and the coefficients and also points on the third level are updated in each iteration step. For the algebraic forecast of any next level points a suitably weighted combination of normals to the locally closed collection of triangles is used respecting visibility constraints.

The procedure will be illustrated starting from a rather coarse, non clustered surface mesh around a generic body with extremely sharp edges and combined convex, concave corners with a need for both, control and smoothing.

References

[1] P. Niederdrenk, “On the Control of Elliptic Grid Generation”, Proc. 6th Intern. Conf. Numerical Grid Generation in Comput. Field Simulations, London, July 6 - 9, 1998, ISBN 0-9651627-2-9