INTEGRATING QUALITY MEASURES WITH ELEMENT APPROVAL

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The quality of the given element in a triangulation is assessed at least twice, once when the element is deemed acceptable by the actual mesh generation algorithm and again when the overall quality of the mesh is measured during a post-processing operation. The quality measures used differ at both stages. Indeed, if they were the same, post-processing would be meaningless. In this paper we investigate augmenting the standard circumcircle radius measure with rotation invariant norms as quality measures at the triangulation level. Our initial motivation was to aid the post-processing by finishing the mesh generation with necessary quality information already available.

Rotation invariant norms, such as Frobenius norm, address well the question over the shape of a triangle[1]. However, elements must adhere to size requirements also. Therefore, one cannot simply replace the standard radius-based measures but one can augment them by emphasising different measures at different stages of the mesh generation. In essence, this approach leads to a problem of finding an optimal path in a two-parameter space with element size and shape as the axes.

The naive, "straight line"-approach is to use the shape as the initial ordering criterion and the radius as the stopping criterion, that is, the triangle with largest deviation from an equilateral one is chosen first for new point insertion, but once every circumradii falls below a threshold level, the algorithm stops regardless of the shapes of the non-accepted triangles. When applied to our suite of Voronoi-Segment algorithms[2,3], this scheme already resulted in improved angle distributions and furthermore, the effect of the relaxation parameter determining how much the size (in terms of radius) can vary, diminished considerably. Our interpretation is that well-shaped element pack together tighter and therefore the number of triangles accepted by their size only is smaller. Similar results were observed using standard Voronoi-Vertex approach. All cases were run without post-processing.

We are currently evaluating more advanced or "non-linear" schemes. For very large meshes one should take the additional computational cost into account also. However, currently the naive scheme is the most robust one.

References

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