EFFICIENT SPACETIME MESHING WITH NONLOCAL CONE CONSTRAINTS

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We consider the problem of efficiently constructing spacetime meshes for spacetime discontinuous Galerkin solutions of nonlinear conservation laws. Our algorithm is an extension of the 'Tent Pitcher' algorithms of Üngör and Sheffer [4] and Erickson *et al.* [1]. We use an advancing front method to construct a spacetime mesh of arbitrary duration over an arbitrary, simplicially meshed spatial domain. Elements are repeatedly added to the evolving mesh in small patches by moving a vertex of the front forward in time. In order to support independent solutions within each patch, no point on the outflow boundary of a patch can lie inside the cone of influence of any other point on the front. This requirement limits the duration of each patch as it is created. For linear problems, where the wave speed is constant throughout spacetime [5], the height of a patch can be computed by looking only in the local neighborhood of the vertex being lifted [1,4]. For non-linear problems, however, the duration of a patch can depend on fast wave speeds arbitrarily far away in space.

To allow us to compute the most limiting cone of influence quickly, we maintain a *bounding cone hierarchy* for the advancing front. Our approach is similar to common techniques for collision detection in computer graphics [3] and for overlaying planar maps in geographic information systems [2]. The hierarchy is defined by a recursive spatial decomposition, such as a quadtree, of the underlying spatial domain. For each region in this decomposition, we maintain a conservative approximation of the cone of influence for that entire region. We can compute the most limiting cone constraints at any point in spacetime by adaptively searching through this hierarchy. In light of the efficiency of related methods [2,3] and our own preliminary experiments, we expect this search to be extremely efficient in practice. Each time we create a patch, we can update the cone hierarchy in $O(\log n)$ time, where n is the number of nodes on the front, by traversing a single path from a leaf up to the root.

References

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