# A NEW NODAL MOVING LEAST-SQUARE METHOD IN FINITE ELEMENT APPROXIMATION SETTING

**Yong Gie Choi**[a] and **Byung Man Kwak**[b]

Dept. of Mech. Engng., Korea Advanced Institute of Science and Technology
373-1 Guseong-dong Yuseong-gu, Daejon 305-701 Korea
[a]cyg@khp.kaist.ac.kr
[b]bmkwak@khp.kaist.ac.kr

In general, domain integration in common Galerkin mesh-free methods cannot be exactly calculated even by using a high order Gauss quadrature integration because of very complex non-polynomial shape functions. The integration error causes inaccuracy of solution and failure of patch test although the mesh-free shape function can reproduce linear fields[1,2]. To overcome the weakness, several methods, such as stabilized conforming nodal integration[1] and least-squares meshfree method[2], have been suggested.
In this paper, we propose a new solution approximation technique, embodied in a combination of finite element and moving least-square method frequently used in mesh-free methods. The proposed scheme is named "the nodal least-square approximation method(NLSAM)." NLSAM passes high order patch test.
The proposed approximation $u^h$ of a variable $u$ is

$$u^h(\mathbf{x}(\xi)) = \mathbf{p}(\mathbf{x})^T \mathbf{a}(\xi) \tag{1}$$

$$\mathbf{x}(\xi) = \sum_I \mathbf{x}_I N_I(\xi) \tag{2}$$

$$\mathbf{a}(\xi) = \sum_I \mathbf{a}_I N_I(\xi) \tag{3}$$

where $\mathbf{p}(\mathbf{x})$ is the vector of all basis polynomials of order less than or equal to $m$, $N_I(\xi)$ is a finite element shape function and $\mathbf{a}(\xi)$ a coefficient vector.
We find the unknown coefficient vector $\mathbf{a}_I$ at each node $\mathbf{x}_I$ by using the moving least-square procedure at the corresponding node to minimize the localized approximation error functional, $J(\mathbf{a}_I) \equiv \| u^h - u \|^2_{\mathbf{x}_I}$.
In the proposed method, the reproducing property for $\mathbf{p}(\mathbf{x})$ is satisfied even under an irregular node distribution if the $N_I(\xi)$ is a partition of unity, $\sum_I N_I(\xi) = 1$. It is notable that not only the proposed shape functions but also its derivatives with respect to $\mathbf{x}$ are piecewise simple polynomials if simplex linear finite elements(i.e. 3-node triangles for 2-D or 4-node tetrahedrons for 3-D) are used for $N_I(\xi)$. This means the domain integration in the Galerkin method can be exactly calculated by a suitable domain integration scheme in a linear problem. Thus the proposed method passes not only the first order but also a high order patch test for all basis polynomials, $\mathbf{p}(\mathbf{x})$.
In general, the construction of the simplex finite element mesh, required in the proposed method, from a CAD model or randomly scattered node set is not expensive, and the mesh can be directly used for domain integration without an additional construction of integration cells required in usual mesh-free methods. This makes the proposed method very attractive and easily applicable to real problems.

## References

[1] J. S. Chen, C. T. Wu, S. Yoon and Y. You, "A Stabilized Conforming Nodal Integration for Galerkin Mesh-free Methods," *International Journal for Numerical Methods in Engineering*, v. 50, pp. 435-466, 2001.
[2] S. H. Park and S. K. Youn, "The Least-squares Meshfree Method," *International Journal for Numerical Methods in Engineering*, v. 52, pp. 997-1012, 2001.