ADVANCES IN ENRICHED FINITE ELEMENT APPROXIMATIONS : INTERPRETATIONS OF VARIATIONAL CONSISTENCY

J.E. Dolbow

Department of Civil and Environmental Engineering
Duke University
Durham, N.C. 27708-0287
jdolbow@duke.edu

Recently, much attention has been focused on a class of finite element approximations that allow for the representation of sharp interfaces and discontinuities on fixed meshes. Beginning with the work of Belytschko and Black [1], these methods enhance the approximation with enriched basis functions that evolve with the feature of interest. Despite their potential, a number of obstacles to the wider implementation of enriched finite element approximations can be identified. For example, it is well known that the usual error estimates for finite elements break down in the vicinity of discontinuities. The conventional wisdom is that such estimates can be recovered simply by allowing the trial functions to mimic the jump conditions across the discontinuity surface. In the above context, this translates to the use of enrichment functions with *slope discontinuities*. While it is clear that a wide range of enrichment functions with this property can be constructed, it is also clear that not all such functions result in the same global rate of convergence [2]. Along completely different lines, fairly little work has addressed the potentially limiting issue of satisfying volumetric incompressibility constraints with these approximations. Recent studies indicate that the standard incorporation of enrichment within locking-free elements can result in spurious stress oscillations on surfaces of discontinuity [3].

In this two part presentation, we invoke a broad interpretation of variational consistency to address the above issues. In the first part, we demonstrate how standard domain integrals can be adapted to evaluate arbitrary jump quantities across material interfaces. The new strategies are shown to be critical to simulate phase transitions in material systems governed by coupled stress and diffusion, wherein a minimum of bulk dissipation is available to smooth local errors. In the second part, we present a new approach for enriching enhanced assumed strain elements. An argument of variational consistency is invoked to construct enhanced strain functions that properly interact with the richer space of functions spanned by the gradient of the enriched displacement approximation [3]. In particular, in those elements intersected by a failure surface, we construct enhanced strain approximations which are orthogonal to *piecewise-constant* stress fields. Contrast is drawn with existing strong discontinuity approaches where the enhanced strain variations in localized elements is constructed to be orthogonal to constant nominal stress fields.

References

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[3] J. Dolbow and A. Devan, "Enrichment of Enhanced Assumed Strain Approximations for Representing Strong Discontinuities : Addressing Volumetric Locking and the Discontinuous Patch Test," *International Journal for Numerical Methods in Engineering*, accepted for publication, 2003.