MULTI-SCALE SIMULTANEOUS ANALYSIS INVOLVING MATERIAL INSTABILITY INDUCED BY MICROSTRUCTURAL BIFURCATION

I. Saiki[a], **K. Ooue**[b], **K. Terada**[c] and **A. Nakajima**[a]

[a]Department of Civil Engineering, Utsunomiya University
Yoto, Utsunomiya, Japan 321-8585, saiki@cc.utsunomiya-u.ac.jp

[b]Development Division, Kawada Technosystem Co., Ltd.
Arakawa, Tokyo, Japan 116-0014

[c]Department of Civil Engineering, Tohoku University
Aoba, Sendai, Japan 980-8579

In the multi-scale method based on the mathematical homogenization thory, the bifurcation in the microscale problem can be regarded as a kind of material instability in the sense that it allows macroscopic, i.e. homogenized constitutive equation to have multiple solutions. In this context, authors have established a computational method to characterize material instability by means of the microstructural instability [1]. In this paper, we present multi-scale simultaneous analysis of solids with periodic microstructures undergoing bifurcation on the basis of the mathematical homogenization.

For this purpose, difficulties due to the loss of convexity of the homogenized potential energy is overcome by the variational formulation based on the Γ-convergence theory [2]. Moreover, minimization of homogenized energy with respect to the number of unit cells is achieved with the help of block-diagonalization method [1]. Finally, we propose an algorithm for multi-scale simultaneous analysis, which involves the interaction of instability phenomena in macro- and micro-scale problems, consistent with derived multi-scale variational formulation. Nature of the material instability induced by bifurcation in microstructure is also discussed within the framework of the two-scale boundary value problems proposed here.

The representative numerical examples of macro- and micro-scale simultaneous analysis are demonstrated for some potential systems which microstructures undergo the bifurcation. These examples show the feasibility of the proposed multi-scale simultaneous analysis and illustrate the macroscopic bifurcation phenomena associated with the microstructural ones.

References

[1] I. Saiki, K. Terada, K. Ikeda and M. Hori, "Appropriate number of unit cells in a representative volume element for micro-structural bifurcation encountered in a multi-scale modeling", *Comp. Meths. Appl. Mech. Eng.*, Vol. 191, p. 2561-2585, 2002.

[2] S. Müller, "Homogenization of nonconvex integral functionals and cellular elastic materials", *Arch. Rat. Mech. Anal.*, 99, p. 189-212, 1987.